WHO PAYS FOR SWITCHING COSTS?

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ABSTRACT. Earlier work characterized pricing with switching costs as a dilemma between a short-term "harvesting" incentive to increase prices versus a long-term "investing" incentive to decrease prices. This paper shows that small switching costs may reduce firm profits and provide short-term incentives to lower rather than raise prices. We provide a simple expression which characterizes the impact of the introduction of switching costs on prices and profits for a general model. We then explore the impact of switching costs in a variety of specific examples which are special cases of our model. We emphasize the importance of a short term "compensating" effect on switching costs. When consumers switch in equilibrium, firms offset the costs of consumers that are switching into the firm. If switching costs are low, this compensating effect of switching costs causes even myopic firms to decrease prices. The incentive to decrease prices is even stronger for forward looking firms.

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1. Introduction

Switching costs (SC) are a common feature in markets where consumers make repeated purchases over time. Starting from Klemperer (1987), the economic analysis of SC compared a short term incentive to "harvest" loyal customers with a long term incentive to attract new customers ("invest") for future harvesting. This is echoed in the recent handbook chapter by Farrell and Klemperer (2007) who conclude that the majority of theoretic models imply that the short term effect dominates and SC should relax competition and increase prices.

This paper highlights an additional short term effect of SC beyond the harvest-invest dichotomy that actually decreases prices. If most of the firms' marginal customers pay the switching cost, an increase in the SC acts like a tax. The firm has to decrease prices below the competitive level in order to *compensate* the customers that are switching from other goods. This downward pressure on prices applies even if the firm is completely myopic and has no incentive to invest and increase its market share for future harvesting.

The *compensating* effect of SC is the mirror image of the harvesting effect. Whereas harvesting reflects the ability to charge more to loyal customers—who are less likely to switch due to SC—compensating reflects the short-term value of non-loyal customers who can be induced to switch products. The compensating effect is distinct from the long-term investing effect because it reflects the short term value of marginal customers to today's revenue while the investing effect reflects the long term value of a loyal customer tomorrow who will be willing to pay more for the firm's good in order to avoid any switching costs.

Whether an increase in SC generates a short term incentive for the firm to increase or decrease prices depends on the balance between the compensating and harvesting effects. This, in turn, depends on the ratio between the firms' marginal customers that would have to pay the SC and those that do not. Our results show that in many situations when SC are low, the compensating effect by itself explains why SC decrease resulting prices and profits, even if firms are completely myopic.[1] In particular, attributing the decrease in prices only to long run investing behavior by the firms in such settings, as is suggested by the harvest-invest approach, is unwarranted.

While intuitive, common modeling assumptions obscure the compensating effect. First, much of the earlier work (e.g., Farrell and Shapiro, 1988; Beggs and Klemperer, 1992) assumed that SC are so large that, at least in equilibrium, all of the firms' marginal customers are non-switchers. This eliminated the compensating effect by construction. Our results, in contrast, focus on the case that SC are low. A second alternative assumption is price-discrimination based on consumer loyalty (e.g., Chen, 1997; Dube, Hitsch and Rossi, 2009; Cabral, 2012). While the qualitative result under price discrimination is similar to ours—SC create a short term incentive to decrease prices—the economic forces leading to it are different. In particular, firms can compete fiercely for

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[1] Many empirical studies consider small SC (e.g., Keane, 1997; Shy, 2002; Shcherbakov, 2010; Dubé, Hitsch and Rossi, 2010; Pavlidis and Ellickson, 2014). We discuss these in more detail below.

customers that are not loyal to them without affecting the profits from their loyal customers. This changes the short term game considerably from the non price-discrimination case and replaces the compensating effect. In particular, the optimal prices under price discrimination are independent of the relative size of each loyalty group, which is not the case without price discrimination. Finally, a common assumption is that all firms in the market are symmetric and there is no outside good (e.g, Klemperer, 1987; Rhodes, 2014).[2] Proposition 3.1 shows that in such settings, if consumers are myopic and SC are low, the short term effects (harvesting and compensating) cancel and prices and profits decrease due only to the investing effect. Our results show that this result is knife-edge, and for example, can be reversed by the introduction of an outside good.[3]

The model presented in this paper—small switching costs in competition between asymmetric firms that do not price discriminate—is applicable to understand the dynamics of many consumer markets which have been studied empirically (e.g., orange juice, yogurt, ketchup – all considered in the empirical literature cited below).[4] This paper is the first to formalize the short term *harvesting* and *compensating* incentives as well as the long term *investing* incentive of SC using an infinite-horizon framework. While it is possible to see the compensating effect even in a static game, the dynamic setting is required to determine the firms' steady state market shares and the resulting balance between harvesting and compensating in equilibrium (Klemperer, 1987). This holds even if firms are myopic. In addition, a static analysis cancels the important investing effect for forward looking firms.

To determine the conditions under which switching costs are likely to increase or decrease prices, we examine a series of special cases of our model. Under surprisingly modest conditions the compensating effect dominates short-term incentives and causes even a myopic firm—seeking to maximize only present-period profits—to react to an increase in SC by lowering prices, in contrast to common intuition.[5] The more familiar role of the investing incentive is also clear, as firms become more patient, holding all else equal, they are more likely to decrease prices as a result of the introduction of switching costs. Combining both short and long term incentives, Propositions 3.1 through 3.5 provide precise characterizations of common settings in which the overall effect of SC is a decrease in prices.

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[2]Klemperer (1987) considers several alternative two-period models. Of these, the closest to ours is when consumer preferences are uncorrelated between the two periods. In this setting he focuses on symmetric firms and finds that the short term (second period) effect is zero and the first period (investing) effect decreases prices and profits. Our analysis generalizes this result and confirms Klemperer's conjecture (in the conclusion there) that in asymmetric markets a firms' reaction depends on its market share.

[3]Pearcy (2014) shows that allowing for consumers to be forward looking can also reverse the qualitative effect of SC on symmetric markets.

[4]Aside from small SC, the main assumption in our analysis is that the consumers' purchasing decision is myopic. With the exception of Pearcy (2014), this assumption is common to the previous studies mentioned above, and relaxing it is an important avenue for future research.

[5]Roughly this result requires that the firm has less than average market share.

To see why the compensating effect is likely to dominate harvesting, consider an environment without switching costs. With many firms in this market, consumers switch frequently, so a firm's marginal consumer is unlikely to be a repeat purchaser. If a low SC is introduced to the market, demand from the "non-loyal" consumers drops while demand from loyal consumers increases. Critically, the firm's prices and profits are still driven by the demand from marginal consumers who in many cases are primarily non-loyal. Thus, compensating dominates: the short term effect of an increase in SC is to reduce prices and profits. Note that this logic holds even if firms are myopic (i.e., there is no incentive to invest). If firms are forward looking, the investment effect provides additional incentive to lower prices.

An important implication of our analysis is that price changes may not reflect the welfare effects of SC. An increase in SC will generally reduce consumer welfare—consumers are either forced to pay an extra SC or to stay with a good they would rather switch away from. However, an increase in SC may either decrease or increase prices and profits. We find that, in competitive markets with small SC, an increase in SC is likely to decrease consumer welfare, prices and profits.

Our analysis adds to a long literature on SC.[6] Farrell and Klemperer (2007) provide a detailed and current review of the economic analysis of SC, outlining the "harvesting" and "investing" conclusions described above. In addition to the papers mentioned above, the working paper version of Viard (2007) provides an alternative two period model of SC, but the results rely on price competition. Our analysis shows that all the effects occur even if only one firm changes its price in response to the SC while the other firms' prices are fixed, as well and in a more general dynamic setting.

Our work, which does not rely on strong functional form assumptions, complements recent work which investigates switching costs without price discrimination in more stylized models. Recently, Biglaiser, Cremer and Dobos (2013) have pointed out that when SC are heterogeneous, they may result in lower firm profits. This result, however, relies on the existence of "shoppers" who face relatively low (or non-existent) SC, while in our analysis lower profits are obtained even if all consumers face the same SC. We consider heterogeneous SC in Appendix B. Garcia (2011) uses a continuous time framework and a particular functional form for consumer utility and also finds that prices are decreasing in the magnitude of SC. In a commentary, Shin et al. (2009) analyze two-period model of a symmetric Hotelling competition and also find that a small SC would decrease prices. Rhodes (2014) extends the analysis to an infinite horizon setting and finds a closed form solution that confirms the result in Shin et al. (2009). Somaini and Einav (2013) extend a Hotelling-style model with switching

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[6]In this paper, we consider so-called "transactional" switching costs, which are most commonly addressed in the literature. In this framework, a cost must be paid each time consumers switch products. Nilssen (1992) contrasts this type of switching costs with "learning" switching costs, where customers may costlessly switch between products they have already learned how to use.

costs to more than two firms and use their model to analyze the price and welfare effects of mergers in markets with SC. Pearcy (2014) extends the analysis for symmetric firms with large SC and forward looking customers.

Several empirical studies have emphasized that switching costs play an important role in a variety of industries, and have found estimates of switching costs that are consistent with our assumptions of non-zero but non-infinite SC.[7] Shy (2002) uses a homogeneous goods model to approximate SC. In the Israeli mobile phone industry, he finds SC are roughly equal to the cost of a new phone. In the Finnish demand-deposit banking industry, he finds SC are approximately 10 percent of deposits. Shcherbakov (2010) estimates SC in the paid TV industry to be $149 for cable subscribers and $238 for satellite subscribers using market level data. These costs are non-trivial, but are less than the price of a year of satellite or cable television service. Keane (1997) employs a series of models with increasingly flexible forms of unobserved heterogeneity to estimate SC in consumer preferences for ketchup. While SC remain significant after controlling for unobserved heterogeneity, he finds that they are quantitatively small—equivalent to roughly 5 cents. Dubé, Hitsch and Rossi (2010), use consumer-level data and find that the median consumer exhibits SC in retail margarine and orange juice industries on the order of 12% and 21% of product prices respectively. Their results, like Keane's, are robust to highly flexible controls for unobserved heterogeneity and auto-correlation in taste shocks. Pavlidis and Ellickson (2014) separately estimate SC associated with parent brand and sub-brand in Yogurt. They find that both SC are significant and that a reduction in the SC increases prices. In all these cases, consumers have a reasonable probability of switching, which indicates that the compensating effect may be important. Based on our numerical simulations, the level of SC found in these studies are well within the range we might expect to cause price declines relative to the no SC case. While the above studies take a structural approach, an alternative methodology examines plausibly exogenous changes in switching costs to ascertain their effect. Viard (2007) uses a difference-in-difference estimation on the impact of number portability in the 800-number service market. Viard argues SC would be very high without number portability, since firms use their 800-number in marketing campaigns. He finds that the introduction of number portability (a substantial reduction in SC) caused prices to fall. Park (2011) considers the impact of number portability in the retail cellular phone market. Park finds evidence that the introduction of number portability reduced prices for high-volume consumers—who presumably had high SC—but not for low-volume consumers who would be likely to have low SC.

The next section presents the model and the abstract result. Section 3, derives the SC comparative statics when SC are small for various demand specifications. Finally, we augment the analysis with numerical simulations

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[7]Klemperer (1995) argues that SC are likely to play an important role in many areas of economics, including industrial organization and international trade.

that confirm the effect for empirically relevant parametrizations. Section 4 summarizes our results and briefly discusses implications for policies aimed at reducing SC. All proofs are provided in Appendix A.

2. A Switching–Costs Model

2.1. The Basic Model. Time is discrete and infinite, indexed by $t \in \{0, 1, 2, \ldots\}$. There are J goods, each produced by a different firm. We allow for "inside" and "outside" firms. The only difference between the two is that while all inside firms j announce each period a price $p_{t,j}$, the outside firms' price is assumed to be exogenous to the analysis. The vector of period t prices is $p_t = (p_{t1}, ..., p_{tJ}) \in R^J$.

The demand model generalizes the commonly used Logit and Hotelling models. Assumption 2.1 provides the formal description. There is a unit mass of consumers, all of which have the same preferences over products and money up to preference shocks and switching costs. To capture demand heterogeneity, each period each consumer receives an i.i.d. (over consumers, goods and time) preference shock. Consumers purchase each period the good that maximizes their utility net of price. As usual, integrating out these shocks provides a demand function for each good that depends on the distribution of the shocks. Since the shocks are i.i.d. a consumer that bought good j in period t may prefer another good in period $t + 1$. This is the source of switching in the model.

Each consumer belongs to a specific loyalty group. A consumer can only be loyal to one good at a time and becomes immediately loyal to the last good purchased. Consumers in loyalty group j (i.e., who purchased j last period) derive $\gamma \geq 0$ more utility from j than an otherwise identical consumer who is in some other loyalty group (i.e., did not purchase j last period).[8] Thus, γ is the switching-cost value. For the consumer, the cost of a good is higher by γ if the consumer has to "switch-into" that good. If $\gamma = 0$, then there are no SC and preference is unaffected by loyalty.

The unit mass of consumers is thus separated to J segments of size s_{tj} defined by loyalty to good j. The vector of market shares, $s_t = (s_{t1}, \ldots s_{tJ})$, identifies the state of the market. We restrict attention to Markov pricing strategies by the firms (i.e., firm strategies depend only on s_t).

The key primitive for the analysis will be the induced demand function for good j at period t within each loyalty group k, denoted $D^{t,j,k}$. That is, $D^{t,j,k}$ is the share of consumers that bought good j in period t out of the set of consumers that bought good k in period $t - 1$. We drop the period indication from the notation unless more than a single period is referenced.

The following summarizes the demand assumptions:

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[8]Equivalently, one could normalize utility without loss of generality such a consumer in loyalty group j derives γ less utility from all goods except j.

Assumption 2.1. Of the customers loyal to good k at the start of the period, the share that choose good j depends only on the current prices p and the SC parameter γ. This share is given by the demand function $D^{j,k}(p, \gamma)$ which satisfies the following:

(1) When switching costs are not present, $\gamma = 0$, the share of of consumers purchasing good j is the same for all loyalty groups of previous purchasers. That is, for all price vectors p,

$$D^{j,k_1}(p, 0) = D^{j,k_2}(p, 0)$$

This ensures that consumers preferences are only affected by previous purchases only through the switching cost. It also rules out any other sources of consumer preference heterogeneity that would be correlated with previous purchasing decisions.[9]

(2) Demand is smooth in prices, decreasing in own price and increasing in rival prices:

$$\frac{\partial D^{j,k}(p, \gamma)}{\partial p_j} \leq 0 \leq \frac{\partial D^{j,k}(p, \gamma)}{\partial p_{i \neq j}}$$

(3) An increase in the SC increases (decreases) demand for a good from it's loyal (switching) customers:

$$\frac{\partial D^{j,j}(p, \gamma)}{\partial \gamma} \geq 0 \geq \frac{\partial D^{j,k \neq j}(p, \gamma)}{\partial \gamma}$$

Note that because $D^{j,j}$ represents the share of consumers repurchasing, this inequality is intuitive whether the direct effect of γ is to increase the benefit of the re-purchasing j (habit formation) or to decrease the utility of having to switch to another product (switching cost).

The two demand specifications that are most common to studies of SC are "logit with SC" (e.g., Dube, Hitsch and Rossi, 2009; Pearcy, 2014) and "Hotelling with SC" (e.g., Rhodes, 2014). Both satisfy all of our assumptions. Section 3 considers the logit model in detail. Appendix C provides similar qualitative results this for the Salop (1979) circular city model that generalizes the Hotelling (1929) model.

Assumption 2.1 reflects three economic assumptions. First, it implies that consumers are myopic: demand does not depend on any future expectations on prices. We make this assumption for tractability reasons, as introducing forward looking consumers make demand dependent on consumer's price expectations, which would need to be determined in equilibrium. Second, we assume that demand changes smoothly in prices and switching costs. This rules out out Bertrand-trap pricing behavior. Finally we assume that other sources of consumer heterogeneity aside from switching costs are idiosyncratic over time. This allows us to compute shares for each group without explicitly tracking the distribution of tastes within a group into account, as they do not depend

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[9]In particular, this rules out network effects.

on previous purchases. In a more general model with time persistent consumer heterogeneity, consumers with a persistent preference for a good would be over-represented in that good's loyalty group even when switching costs were very small. It is natural to expect that consumers that choose product j do so in part because of some persistent preference to product j's characteristics. We consider this extension in Appendix B. To the extent that time-persistent sources of consumer heterogeneity decrease the competitiveness of the market, it strengthens the harvesting effect of SC and weakens both the compensating and investing effects.

We now turn to the firms' pricing game. Total demand for a firm is the aggregate of group specific demands,

$$(2.1) \qquad D^j(s, p, \gamma) = \sum_{k=1}^{J} s_k D^{j,k}(p, \gamma)$$

The state variable for this model is the vector of market shares. The law of motion is simple: Firm j's total demand in period t is also the measure of consumers that will be loyal to firm j at the start of the next period:[10]

$$(2.2) \qquad s_{t+1,j} = D^j(s_t, p_t, \gamma)$$

Each firm maximizes profits over an infinite horizon given a common discount factor, β. We assume that the firms have identical marginal costs normalized to zero.[11] The present discounted value of firm j's future profits given the pricing strategies p are

$$\Pi_j = \sum_{t=0}^{\infty} \beta^t p_{jt} D^j(s_t, p_t, \gamma)$$

Subject to the law of motion specified in (2.2). The following assumption on the equilibrium outcome *without* SC is technical and shared by virtually all relevant demand models. It ensures that the comparative static of introducing SC is well-defined.

Assumption 2.2. If there are no SC ($\gamma = 0$), the period profit function, $p_j D(p, s)$, is quasi-concave in p_j and strictly positive for all firms. The game when $\gamma = \beta = 0$ has a unique equilibrium.

If $\beta > 0$, the firms play a dynamic game. When SC are absent ($\gamma = 0$), the game is a standard repeated game and the folk theorem applies.[12] Adding SC does not alter the possibility of collusion. The multiplicity of equilibria makes it difficult to analyze the impact of SC. We focus only on the "competitive" Markov Perfect Equilibrium. When SC are zero, there is no future value to gaining market share, and the most competitive equilibrium is simply to price to maximize stage game profits in every period. Assumption 2.2 ensures that this

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[10]It is also possible to analyze alternative laws of motion that might allow for the introduction of new consumers, or experimentation by consumers who might try a new good, but remain loyal to their earlier purchase with some positive probability. Both of these extensions have the effect of reducing firms' investment incentive, but do not change the qualitative results of our model.

[11]This normalization is for convenience and the model can be easily generalized to accommodate firm-specific marginal costs.

[12]Moreover, there is no reason to believe that restricting ourselves to stationary strategies will ensure a unique equilibrium as firms may detect defections from market shares.

"competitive" equilibrium is unique. Under this scenario, the firms name the same prices in each period regardless of last period market share, and it is clear that this equilibrium strategy will converge to a steady state. We analyze the effect of SC by perturbing this competitive equilibrium to characterize the change in steady state prices, profits, and share with due to the addition of SC.

2.2. **Two Fundamental Equations.** This section establishes the economic forces that determine the comparative static effects of small SC on profits and prices.

By the Principle of Optimality, the firm's problem around the steady state can be represented as the solution to the following Bellman equation for firm j, where $V^j(\cdot)$ is a stationary function of the current state s and $D(\cdot)$ is the vector of the resulting demand for all firms.

$$(2.3) \qquad V^j(s;\gamma) = \max_{p_j} p_j \cdot D^j(s,p,\gamma) + \beta V^j(D(s,p,\gamma);\gamma)$$

We wish to characterize the behavior of the firm in a steady state and derive comparative statics with respect to the introduction of SC. The definition of a steady state is standard—all firms optimally set prices and consumer purchasing behavior is such that the states (i.e., the firms' market shares) are constant over time. Note that while shares are constant, many consumers may be switching goods between periods.

We first consider the impact of an introduction in switching costs on firm values. The effect relies on partial effects of prices, shares and the SC themselves on demand at the steady state without SC. These are all well defined via Assumption 2.2. While these are abstract at this point, they are all well known for any demand specification. The proof is provided in the Appendix A.

Lemma 2.1. *The derivative of firm j's steady-state value with respect to SC is given by*

$$(2.4) \qquad \frac{dV^j}{d\gamma} = \frac{p_j^*}{1-\beta} \cdot \left[D_\gamma^j + \sum_{k \neq j} \left(\frac{dp_k^*}{d\gamma} \cdot D_{p_k}^j \right) \right],$$

where p^* represents the vector of steady state prices for the equilibrium when $\gamma = 0$.

Lemma 2.1 identifies the two forces determining the qualitative effect of SC on firm profits. Simply put, SC are good for the firm if they increase demand for the firm, and bad if they decrease it. There are two channels for SC to affect demand. First, holding prices fixed, SC increases demand by loyals and decreases demand by non-loyals. The second, indirect effect is the competition effect—if SC cause rivals to reduce prices, this will affect demand as well. As we will show below, in markets with no significantly dominant firm,[13] both effects will be negative with small SC, leading to the conclusion that SC hurt competitive firms.

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[13]The propositions below formalize dominance using bounds on market shares and HHI.

As all policy functions and state transitions are smooth, the first order condition for V^j with respect to p_j is

$$(2.5) \qquad F^j = D^j + p_j D^j_{p_j} + \beta \frac{dV^j}{d\nabla_{p_j}} \frac{d\nabla_{p_j}}{dp_j}.$$

$D^j_{p_j}$ is the derivative of the firm's demand w.r.t. it's price and ∇_{p_j} is the gradient of the change in demand for all firms resulting from the firm's price change. The first order condition has no closed form solution in general, and is difficult to characterize due to the effect of changes in switching costs on firms implicitly defined continuation values. To make progress, we will focus on the effect of small SC—in particular, the effect of an increase in SC from zero. This case makes it possible to characterize the continuation value term. To simplify the notation, we let $D^{j,i}_\gamma$ denote the change in firm j's demand among loyals to good i due to the change in SC, holding all else fixed, and let D^j_γ denote the total change for firm j aggregating over all loyalty markets:

$$D^j_\gamma = \sum_{k \in J} s_k D^{j,k}_\gamma.$$

For any specific demand function, D^j_γ can be derived directly at $\gamma = 0$, as we show for the demand specifications considered in the following section.

The equilibrium price effect of the introduction of SC relies on the first order effect of SC on each firm's price, as well as the interaction effect between the firm's prices. Our strategy for the general analysis is to focus instead on sufficient conditions for which the price effect can be signed (either negative or positive).[14] For this, it is sufficient to consider SC effect on each firm's first-order condition (2.5), which we refer to as F^j_γ. The next lemma identifies the formal price effect of the introduction of small switching costs. As before, the effect is characterized by equilibrium outcomes from the no switching cost steady state. While these are abstract at this point, they can be derived for any given demand specification. Thus, Lemma 2.2 can be applied directly in many cases, as is done in the remaining sections.

Lemma 2.2. *The derivative of firm j's steady state price with respect to SC at $\gamma = 0$ has the same sign as F^j_γ, given by:*

$$(2.6) \qquad F^j_{\gamma=0} = D^j_\gamma + p^*_j \cdot D^j_{p_j \gamma} + \beta p^*_j \cdot \sum_{k \in J} \left(D^{j,k}_\gamma \cdot D^k_{p_j} \right) + \sum_{k \neq j} \left(\frac{dp^*_k}{d\gamma} \chi_{jk} \right)$$

Where all $\chi_{jk} \geq 0$ and p^* represent the vector of steady state prices for the equilibrium when $\gamma = 0$. Moreover, long run incentives—i.e., $\sum_{k \in J} \left(D^{j,k}_\gamma \cdot D^k_{p_j} \right)$—are always negative.

Equation (2.6) is obtained by manipulating the full derivative of F^j with respect to γ at $\gamma = 0$, holding p_j fixed.

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[14]The exact price effect is derived for a monopolist in section 3.4.

The first two terms of $F^j_{\gamma=0}$ are simple derivatives reflecting the static incentives of switching costs. D^j_γ is the direct effect of SC on demand holding prices fixed through greater consumer inertia. It is a mixture of the "harvesting" effect for loyal consumers ($D^{j,j}_\gamma > 0$) and the "compensating" effect for switching consumers ($D^{j,k \neq j}_\gamma < 0$). If harvesting dominates, the firm's optimal response is to raise prices. However, if most marginal consumers are switchers, SC will depress demand for the firm's good and the firm will have a short term incentive to decrease prices to compensate. The second term, $p^*_j \cdot D^j_{p_j \gamma}$, is a second order effect capturing the change in price sensitivity. Prices increase (resp. decrease) if consumers are less (resp. more) price sensitive with higher SC.[15]

The third term captures the investing effect. When SC increase, j's next period demand increases for customers that bought j's good today and decreases for all other customers. The total future profit effect is $p^*_j \cdot \sum_k D^{j,k}_\gamma$. Multiplying by the price sensitivity obtains the first order investing effect on the optimal price. As expected, the proposition confirms the results obtained in previous related literature – long term (investing) considerations related to SC always decrease prices.

Finally, each element in the last term accounts for j's price reaction to any of it's rivals' price reaction to the SC. The rival price reaction $\frac{dp}{d\gamma}$ includes both short and long term considerations and in general could be positive or negative. The firm's price reaction to the rival's reaction ($\chi_{j,k}$) is evaluated at the no-SC equilibrium and so is positive by strategic complementarity. Therefore, if we find that F^j_γ has the same sign for *all* firms given $\chi_{jk} = 0$, then this is also the sign for the true value of $\chi_{jk} \geq 0$.

Lemmas 2.1 and 2.2 identify the two fundamental equations that determine the effect of small switching costs on prices and profits. These equations show that the effect can be determined by the steady state price and shares *without SC* and the partial derivatives of demand at no SC: D^j_γ, $D^k_{p_j}$, $D^j_{p_j \gamma}$, and $D^{j,k}_\gamma$. By continuity, the signs of $F^j_{\gamma=0}$ and $V^j_{\gamma=0}$ determine the effect of a small SC on firm profits and prices.

The next section applies these general results to analyze the economic forces that determine the effects of SC on prices and profits.

3. Determining the Effect of Small Switching Costs

The previous section provided general results that do not depend on the specific demand structure. To identify the specific determinants for the effect of SC, some structure must be imposed. This section applies the abstract analysis to a quasi-linear discrete choice random utility demand system, and also makes use of its special case, the logit demand model, which is widely used in theoretical and empirical work.[16] We find that under the logit model, a firms' short run price incentive to raise or lower prices is closely related to its share relative to the

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[15]Note that consumers in group x are less price sensitive if $D^x_{p\gamma} \geq 0$.

[16]Similar results for the Salop demand system are provided in Appendix C.

Herfindahl index. We also consider the implications of firm symmetry, which is commonly used for tractability in theoretical analysis. Finally, we introduce an outside good into the model. This in turn enables us to analyze the effect of switching costs on a monopolist. One point that comes out of the analysis is that the important intuitions are gleaned from the firm's first-order reaction to the change in SC—the first three terms of (2.6)—rather than any reaction to the rival's first-order reaction—the last term of (2.6).

The logit demand system is a special case of a discrete-choice random utility model. In a random utility model, consumers have heterogeneous utilities and purchase one unit of the product that maximizes their utility. In our setting, the utility of person i, for purchasing good j when she purchased k last period (and hence is 'loyal' to k) in time t is,

$$(3.1) \qquad u_{i,j,k} = \bar{u}_{j,k}(p_{jt}, \gamma) + \epsilon_{i,j,k,t}.$$

Where ϵ_{ijkt} is the iid preference shock that induces preference heterogeneity among consumers. We will assume that utility is quasi-linear in price and SC, that is,

$$(3.2) \qquad \bar{u}_{j,k}(p_{jt}, \gamma) = \delta_j + \alpha p_{jt} - \gamma \mathbf{1}\left[j \neq k\right].$$

Proposition 3.1 in Section 3.2 will be provided for the general quasi-linear utility case. However, the logit demand system, which is defined by assuming that ϵ_{ijkt} follows the type-1 extreme value distribution, allows for stronger results. Logit demand has been popular in recent empirical and theoretical investigations of SC—e.g. Dube, Hitsch and Rossi (2009) and Pearcy (2014). As is well known, the logit demand model is especially convenient because it gives rise to a a closed form expression for group-level demand:

$$(3.3) \qquad D^{j,k,t} = \frac{e^{\bar{u}_{j,k}}}{\sum_{l \in J} e^{\bar{u}_{l,k}}},$$

which makes it possible to derive precise results. It will be useful to recall the Herfindahl index when $\gamma = 0$ as a measure of industry concentration, defined as:

$$H = \sum_{j=1}^{J} s_j^2.$$

The next lemma identifies the main quantities used in the general Lemmas 2.1 and 2.2 for logit demand:

Lemma 3.1. *With logit demand, at $\gamma = 0$:*

$$(3.4) \qquad D_\gamma^j = s_j \cdot (s_j - H)$$

$$(3.5) \qquad F_\gamma^j = \frac{s_j^2}{1 - s_j} \cdot ((s_j - H) - \beta \cdot (1 + H - 2s_j)) + \sum_{k \neq j} \left(\frac{dp_k^*}{d\gamma} \chi_{jk} \right)$$

Where all $\chi_{jk} \geq 0$.

Again, χ_{jk} incorporates all the effects resulting from any change in a firm k's price on j's incentives, which, by strategic complementarity, is weakly positive. As alluded to in Section 2.2, the demand specification replaces the general terms in Lemmas 2.1 and 2.2 with the steady state prices and shares without SC. The next subsections identify the various economic considerations that determine the effect of SC: long vs short-term firm reaction, symmetric vs. asymmetric firms and the existence and strength of an outside good.

3.1. **Myopic Firms.** The first term of (3.5) identifies the determinants of a myopic firm's reaction to a change in the SC. This reaction is based on the effect the SC has on the marginal consumers' willingness-to-pay (WTP) for the firm's product. Every marginal consumer for firm j is choosing between product j and some other product j' (which may be an outside good). If more marginal consumers making this choice start the period loyal to firm j, then SC shift demand up for firm j for this segment, and allow the firm to increase prices. If more marginal consumers start the period loyal to the firm's rivals, SC shift demand down for firm j and the firm would decrease prices.

Equation 3.5 shows that in the logit model, the correct way to aggregate the myopic effect for a firm over all of it's rivals is to compare the firm's market share against the Herfindahl index. Moreover, as the long term consideration is always to decrease price (Lemma 2.2), the following is obtained:

Corollary 3.1. *In the logit model:*

(1) *If a firm's share is lower than the Herfindahl, its reaction to an increase in SC is to decrease price.*

(2) *A firm's short term (myopic) reaction to an increase in SC is to increase price if and only if the firm's share is higher than the Herfindahl index.*

3.2. **Symmetric vs. Asymmetric Firms.** A common starting point for analysis of SC is the case that all firms are identical and there is no outside good. (see e.g. Rhodes (2014) and Pearcy (2014) for two recent examples). Introducing a small SC does not change the symmetry in the steady state. Building on the intuition from the previous subsection, the share of each firm's marginal consumers for which demand shifts down exactly equals the share of those for which demand shifts up. In addition, any decreased price sensitivity by loyal customers is matched by an equal increase in price sensitivity by potential switchers. For logit demand, the implication is that short term considerations have no effect on the firm's reaction. Formally, the first term in (3.5) cancels: $s_j - H = 0$.

This intuition does not depend on the specific features of the logit demand and can be extended to any quasi-linear random utility model (i.e. for any distribution of ϵ_{ijkt} in (3.1) when (3.2) holds).[17]

While the short term considerations cancel in the symmetric quasi-linear setting, long term incentives decrease prices. This would be exacerbated by strategic price reaction—each firm would decrease prices a bit more. As each firm sells the same amount, just for a lower price, firm profits decrease. Therefore, in symmetric markets SC will decrease both prices and profits.

Proposition 3.1. *In a random utility model with quasi-linear utility, if firms are symmetric ($\delta_j = \delta$), then:*

(1) *If firms are myopic ($\beta = 0$), small SC have no effect on prices and profits.*

(2) *If firms are forward looking ($\beta > 0$), prices and profits decrease from a small SC for all firms.*

Proposition 3.1 confirms that a small SC will change the steady state for the worse (from the firms' perspective) in a symmetric market as long as firms are forward looking.[18]

It may be of interest to consider the effect of a small SC on a symmetric industry that starts out of steady state. That is, all firms are equal without SC, but some firms are endowed with a larger market share than others. Equation 3.5 can then be used to determine the price effect of SC on each firm until the steady state is reached. Initially, firms with a higher than average share would set a higher price while firms with a lower share would set a lower price. Eventually, the market would reach the steady state and all prices will be equal.

A related question is whether the price (and thus profit) effects in the symmetric situation are affected by the number of firms in the market.[19] As the number of firms increases, the likelihood that a firm's marginal consumer in this period would also be marginal in the next period decreases. This decreases the returns to investing in loyal consumers. However, the increase in products may also affect consumer price sensitivity in equilibrium. If consumers are more price sensitive, the return on investment increases. In the logit model, the increase in the number of firms decreases price sensitivity and therefore the two effects agree. This allows us to sign the overall effect:

Proposition 3.2. *In a symmetric logit demand model with SC, if firms are forward looking ($\beta > 0$), the effect of SC on prices strictly decreases in absolute terms when a firm is added to the market. The result applies to all quasi-linear random utility models in which equilibrium price sensitivity ($D_{p_j}^j$) weakly decreases in absolute terms when a firm is added to the market.*

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[17]For other demand structures, the interaction between SC and price sensitivity may not cancel when firms are symmetric. For example, switchers may become more price sensitive while the price sensitivity of loyals may not change—e.g., add a term $-\gamma_p p_{jt} \mathbf{1}[j \neq k]$ to $\bar{u}$ in (3.2).

[18]While the proof relies on the quasi-linearity of utility, Appendix C provides a similar result for linear models.

[19]We thank a referee for suggesting this.

Proposition 3.2 implies that as the number of firms increases in the market, the effect of an increase in SC is less pronounced.

Finally, the fully symmetric case has a knife-edge quality since the short-term effect is exactly zero. Our model allows us to easily consider alternatives once a demand structure is specified. Consider markets in which there is one leader and the other firms are symmetric. In this case, if the market leader is worse off or reduces prices from a small SC then all firms have the same qualitative effect. As competition is in strategic complements, a sufficient condition for all the firms to decrease (or increase) prices is that all firms would decrease (or increase) prices holding their rival price fixed. Moreover, as the goods are substitutes, profits for each firm increase in rival prices. Thus, a sufficient condition for low SC to depress prices and profits is that this outcome is obtained for the leader, holding rival prices fixed.

Proposition 3.3. *Suppose that in the logit demand model, $J \geq 2$, firm 1 has share $s_1 > \frac{1}{J}$ and all other firms have equal shares $s_{j>1} = \frac{1-s_1}{J-1}$. A sufficient condition for all prices to decrease with SC is,*

$$(3.6) \qquad\qquad s_1 \leq \frac{1 + J\beta}{J + J\beta}.$$

Proposition 3.3 identifies both the short term and total effects of a small SC with one market leading firm. The market leader can "harvest" from SC as most of its marginal consumers would avoid the switching cost by choosing it again. The exact opposite is true for all other firms, who must compensate their marginal consumer. However, if the market leader is forward looking, the investment effect drives even the market leader to reduce prices as long as it's leadership isn't too strong. Note that the upper bound in Proposition 3.3 decreases with J and is always at least $\frac{\beta}{1+\beta}$.[20] Thus, if firms are sufficiently patient ($\beta \to 1$) if a market leader's share is less than half, prices will decrease. Moreover, in a duopoly ($J = 2$), a market leader reduces prices whenever its share absent SC is less than .75.

3.3. **The Role of an Outside Good.** The firm's reactions for an increase in SC remain qualitatively the same when the market is not fully covered or an outside good exists (i.e., when not all consumers purchase from some firm, so firm shares need not sum to 1). Because the price reaction for each firm is determined by considering demand at zero SC, for the purpose of the analysis, the outside good is simply like another rival with the simplification that it's price is not set strategically.[21]

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[20]It can be shown that with n symmetric market leaders, the same upper bound applies as $J \to \infty$

[21]We focus on the case in which the SC also applies when switching to/from the outside good. Our base case is reasonable for many industries. For example, switching across cable TV, satellite, IP-TV and outside "broadcast" television requires learning the channel layout, acquiring and installing the necessary equipment, and ordering or canceling the relevant services even when switching to or from broadcast. We briefly discuss the alternative where switching to the outside good does not incur SC at the end of this section.

The primary difference between an outside option and a regular "inside" firm is that the outside option's price is not set strategically. Our analysis highlights the fact that this has no first order effect. Thus, understanding the SC effects for a single firm facing an outside good is qualitatively the same as understanding the SC effects for a firm in a fully covered duopoly.[22]

In a market with symmetric firms and an outside good, the short-term effect of an increase in SC therefore depends only on the strength of the firms in the industry relative to the outside option. If the share of customers choosing the outside good is smaller than the share of each firm, then all the firms in the industry will see an increase in demand following an increase in SC, holding prices fixed. Because each firm has more loyal customers than the outside option does, there are more marginal consumers that SC sways to stay with the firm over switching to the outside option than there are marginal consumers that choose to stay with the outside good over switching to the firm. Thus, if the outside option is weaker than any of the firms, the short term effect of an increase in SC is higher prices and higher profits. If the outside option is stronger, the short term effect is reversed, following the exact same line of reasoning. The long term effect, however, still decreases prices and potentially profits, as in the case without an outside good.

Proposition 3.4. *In the logit model, if all firms are symmetric, $\delta_1 = \delta_2 = \cdots = \delta_{J-1}$, and an outside good has a share s_J, then:*

(1) *Prices and profits **increase** with a small SC if firms are myopic ($\beta = 0$) and $s_J < \frac{1}{J}$.*

(2) *Prices decrease from a small SC whenever $s_J > \frac{1}{J}$ or $\beta \geq 0.077$*

We note that the introduction of a small outside good reversed the result obtained for myopic and symmetric firms, reinforcing the intuition that the symmetric analysis relies in part on a knife-edge relationship.

The above analysis assumes SC apply symmetrically to inside and outside goods. An alternative variation to consider is when SC do not affect consumers switching out of the outside good J. This is 'good news' for all the inside firms, and 'bad news' for the outside firm. In particular, for all $j < J$, D_γ^j increases by exactly $s_J \cdot s^j$ compared to our baseline case. However, the investing effect is weaker now, as firms do not have as large a dynamic incentive to entice consumers away from the outside good ($D_\gamma^{j,J} = 0$). Formally, all the components of (2.6) that deal with the outside good become zero. Both of these effects increase prices and profits. Therefore, with symmetric firms, higher SC may in this setting be profitable for the firms.

3.4. **Single Firm and an Outside Good.** An important underlying theme of the analysis is that the rivals' price reaction is only of second order when understanding the effects of a small SC. However, analyzing multiple firms complicates the analysis and obtaining clear results requires stronger demand assumptions.

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[22]This is illustrated and discussed further using the numerical example in Figure 1.
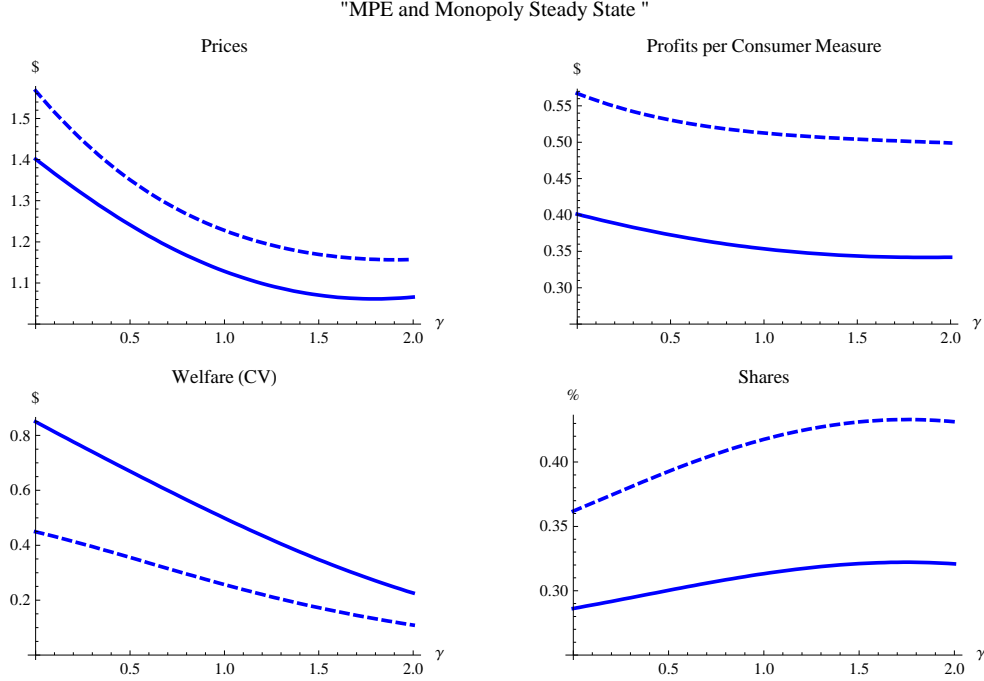
"MPE and Monopoly Steady State "

FIGURE 1. Overall effect of switching costs in the MPE with the monopoly outcomes for reference. Each panel presents a different measure — prices, profits and shares. The thick solid line is the symmetric MPE ($\delta_1 = \delta_2 = 1$, $\delta_0 = 0$). The thick dashed line is the value under a monopoly for the same parameters ($\delta_2 = -\infty$). In all cases, $\beta = 0.95$, $\alpha = -1$ and marginal cost is zero.

Figure 1 strengthens this intuition and the relevance of small SC analysis. The figure presents the logit model steady-state profits, shares, prices and consumer surplus[23] for two settings – a duopoly and a monopoly. In each case, the inside firms' quality is $\delta_j = 1$ and there is also an outside good ($j = 0$).[24] We set the income effect at $\alpha = -1$, firms' discount factor at $\beta = .95$, and the outside good quality δ_0 at cost ($\delta_0 = 0$).

The similarities between the effects of SC for a duopoly and monopoly is striking. The distance between the monopoly and duopoly price and profits (the dashed and solid lines) remain steady for all values of the SC parameter (from $\gamma = 0$ to $\gamma = 2$).[25]

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[23]Consumer surplus is calculated using compensating variation (CV). In the single firm model this is:

$$CV = -\frac{1}{\alpha} \left(\log \left(e^{\delta_0} + e^{\delta_1 + \alpha \cdot p - \gamma} \right) - \delta_0 \right).$$

For high switching-cost values, this method is actually a generous calculation as it assumes that all consumers can stay with the outside good "for free." The main alternative, expenditure variation, (EV) would actually be negative for higher SC as loyal consumers are paying a very high price to keep themselves from switching.

[24]Other symmetric specifications do not add qualitative insight. Specifically, in a symmetric duopoly, no firm can dominate the market regardless of δ. Thus, increasing δ_1 and δ_2 provides no additional insight and is not presented

[25]We use value function iteration to approximate symmetric MPEs. The Mathematica software code is available from the authors. Our simulation used a 361 point grid for the state (share) space and process stops when the change in the value over all states is smaller than a threshold, which was set at 10^{-6}. We verify that the optimal policy is unique by verifying that the best response function is quasi-concave and use the optimal pricing strategy to "play out" the equilibrium strategies starting at various states until a steady state is obtained. A unique steady state was obtained for all parameter values that we consider. For higher SC values, we

This section leverages this insight and analyzes a monopolist's reaction to switching costs in the absence of strategic interaction. This allows us to derive strong results and shows that the effect of switching costs may lead to price declines even when price competition is not present (i.e. a single firm with an outside good).

We maintain the assumption that demand can be derived from a quasi-linear random utility model. Since there is only one firm and an outside good the period utility for consumer i from purchasing the firm's good can be described as,[26]

$$u_{it} = \delta - \alpha \cdot p - \gamma \cdot \mathbf{1}\left[\text{i not loyal to the firm}\right] - \epsilon_{it}.$$

Consumer heterogeneity is captured by ϵ_i, which is distributed i.i.d. across consumers and time, according to some predefined distribution function Φ. The firm's demand is then,[27]

$$D^1 = s_1 \cdot \Phi\left(\delta - \alpha \cdot p + \gamma\right) + (1 - s_1) \cdot \Phi\left(\delta - \alpha \cdot p - \gamma\right)$$

In addition, the following assumption requires that at zero switching costs, the short term effect of SC for the firm considering *only loyal* consumers is to increase price. The requirement is only for the loyal consumers and captures the substance of SC – an increase in demand by loyal consumers. Therefore, it is a very weak and natural assumption. To the best of our knowledge, it holds in all models of SC:[28]

Assumption 3.1. For a monopolist facing an outside good, at $\gamma = 0$, $D_\gamma^{1,1} + p_1 \cdot D_{\gamma p_1}^{1,1} \geq 0$.

The effect of introducing SC on a firm's value and steady state prices is summarized in the following proposition. Recall that p_1 and s_1 are the firm's price and market share in the steady state without SC. All subscripts denote partial derivatives taken using prices and shares at the steady state without SC. [29]

Proposition 3.5. *For a monopolist facing an outside good:*

$$(3.7) \qquad \frac{dV^1}{d\gamma = 0} = \frac{p_1\left(2s_1 - 1\right)}{1 - \beta} D_\gamma^{1,1}$$

$$(3.8) \qquad F_{\gamma=0}^1 = (2s_1 - 1)\left(D_\gamma^{1,1} + p_1 D_{p_1\gamma}^{1,1}\right) - 2\beta s_1 \cdot D_\gamma^{1,1}$$

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do not expect a steady state exists as firms profit from 'invest then harvest' cycles that take advantage of consumer myopia. Indeed, our equilibrium for high SC (roughly $\gamma > 2.7$) did not converge to a steady state.

[26]For expositional simplicity, we subtract ϵ in this section rather than add it as in (3.1). This is without loss of generality.

[27]To see this note that consumers purchase the firm's product if it offers the higher utility, that is the loyal consumer's purchase if $\delta - \alpha \cdot p - \epsilon_i > -\gamma$ and non-loyal consumers purchase if $\delta - \alpha \cdot p - \gamma - \epsilon_i > 0$. Integrating these expressions over ϵ_i leads to the demand equation.

[28]Under the quasi-linear model, the assumption is equivalent to log concavity in Φ, i.e., $(\Phi')^2 \geq \Phi \cdot \Phi''$.

[29]An older version of the paper derived the exact value for $\frac{dp_1}{d\gamma}$:

$$\frac{dp_1}{d\gamma = 0} = -\frac{F_\gamma^1}{2D_{p_1}^1 + p_1 \cdot D_{p_1 p_1}^1}$$

However, this does not provide any additional insights; the proof is available from the authors. Note that the denominator must be negative by the standard second order condition.

Thus:

(1) *A small SC increases profits for a monopolist iff the monopolist share of the market is at least .5.*

(2) *For any discount factor a small SC causes a monopolist to decrease its price if its share is lower than .5.*

(3) *For any discount factor a small SC causes a monopolist to increase its price if in the equilibrium without SC:* $2s_1 \cdot p_1 D_{p_1\gamma}^{1,1} > D_\gamma^{1,1} + p_1 D_{p_1\gamma}^{1,1}.$

Comparing Propositions 3.5 and 3.1 illustrates the connection between the single-firm and multiple-firm analysis. For small switching costs, the impact of SC is closely tied to the firm's market share relative to it's rivals'. In the single-firm case, the rival's share is simply all the remaining share.

The first part shows that switching costs are beneficial to firms who dominate the no-SC market, essentially high quality firms. This is because higher SC increases inertia, which helps large firms but hinders small firms.

Part 2 establishes that firms with low market shares will respond to SC by decreasing prices. The sign of the first element of F_γ^1 depends, as in part 1, only on whether the firm's share is larger than half. The second, investing element is negative, but it's magnitude depends on the discount factor.

The importance of the compensating effect is illustrated when firms are myopic (i.e., $\beta = 0$). In this case, the investing effect necessarily disappears, since it is motivated by the possibility of future profits. However, the introduction of SC will still cause a decline in steady state prices if the firm's share without SC is less than half. This highlights the importance of the compensating effect as an independent countervailing force to the well-known harvesting effect.

We stress that *the price decrease is not caused by any competitor's actions.* It is simply because the marginal consumer is likely switching and thus should be compensated.

Equation (3.8) also shows that the SC makes a more patient firm price lower. This is the investment effect. Raising β affects only the investing incentive of SC, which drives prices unambiguously downward. Combining these two points implies that if a firm is patient, it will never raise prices upon the introduction of switching costs if its market share is less than one half. On the other hand, if a fully patient firm ($\beta \approx 1$) would increase prices, so would a less patient firm.

The intuition for these results is straightforward. If a good has a low value, it will be purchased in any given period by a minority of customers, even absent SC. As the preference to purchase the good is not persistent over time, most consumers that purchase in any given period would not have bought it in the previous period. The introduction of a small SC does not change this "demographic". In every period, most of the firm's buyers are switchers. As a result, SC act like a tax on the marginal consumer and profits and prices are reduced. If the monopolist's initial value proposition to consumers is high, most of the monopolist's consumers are loyal. An

increase in SC makes these loyal customers more likely to buy due to the extra utility from making a repeat purchase. The effect of SC for the monopoly now mimics a subsidy rather than a tax and the monopolist's profits increase.

Applying the analysis to the logit demand used in the previous section:

Proposition 3.6. *For a single firm facing logit demand, the introduction of a small SC changes profits by*

$$(3.9) \qquad \frac{dV^1}{d\gamma = 0} = \frac{p_1 \cdot s_1 \cdot (1 - s_1) \cdot (2s_1 - 1)}{1 - \beta}.$$

The firm's price increases iff $s_1 \geq \frac{1+2\beta}{2+2\beta}$.

Proposition 3.6 makes explicit the abstract intuitions of the general analysis in this section – SC are valuable for a strong firm. Figure 2 illustrates the proposition's implications. The figure plots the derivative of the steady-state price with respect to SC at zero SC for a myopic ($\beta = 0$) and forward-looking firm ($\beta = 0.95$) while varying the product quality, $\delta_1 - \delta_0$. The horizontal axis identifies the quality difference between the firm's good and the outside good, using the optimal firm shares with zero SC.[30] The myopic curve illustrates the balance between the harvesting and compensating effects. The former dominating when the firm share is high and the latter when it is low.

For a myopic firm, the impact of SC on prices turns from positive to negative when the firm has a 0.5 share – precisely when the compensating and harvesting incentives are perfectly offset. The difference between the curves illustrates the investing effect. As an optimal market share of 0.7 without SC is extreme within the logit demand model,[31] the model predicts that prices are likely to decrease in response to small SC whenever the firm is as patient as is typically assumed (i.e. $\beta \in (0.9, 1)$).

The effect of SC on prices is U-shaped with respect to quality (or share). For low quality firms, an increase in quality increases demand elasticity. This increases the magnitude of both the compensating and investing effects. If quality continues to increase, the firm's product is chosen by a larger share of the consumers and eventually demand becomes inelastic – most consumers are infra-marginal. At this point the firm's short term incentive is to harvest. Moreover, as the mass of marginal consumers is very small, the investing effect is weak. This is the state of affairs that most resembles a model of switching costs where consumers become fully 'locked-in' and are unwilling to switch to other products (e.g., Beggs and Klemperer, 1992).

Figure 3 illustrates this relationship between SC, the likelihood of repeat purchase (sometimes referred to as the churn rate) and prices. Panel A of the figure shows that for high SC, the proportion of repeat purchasers increases

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[30]We use steady-state non-SC market shares as a convenient proxy for the underlying firm quality

[31]In a logit setting this requires a quality difference of five, which is over three standard deviations above the mean in the idiosyncratic term. In other words, if the firm would sell at the outside good price, it would have a share of over 0.95.

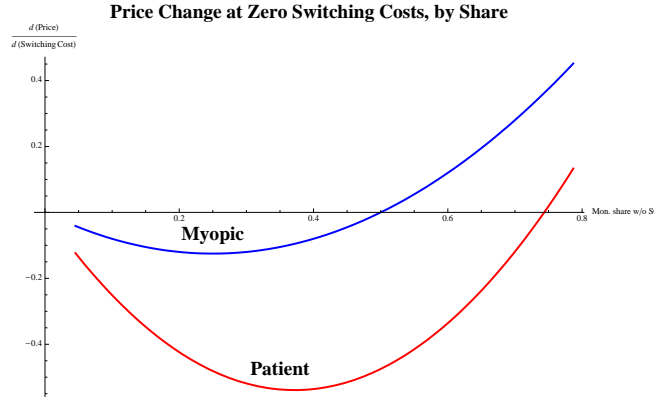**Price Change at Zero Switching Costs, by Share**

FIGURE 2. Derivative of steady-state prices with respect to γ at $\gamma = 0$ for the single firm logit demand model with $\alpha = -1$ and myopic consumers. The x-axis is the firm's market share without switching costs (s_0). The change in market share in the top panel is a reflection only of changing $\delta_1 - \delta_0$. The myopic curve assumes $\beta = 0$ and displays the interaction between the consumer-flow effect and the harvesting effect when there is no investing effect. The patient curve is the same model with $\beta = 0.95$; the difference between the two is the effect of the dynamic marginal value of loyal market share – the investing effect.

as loyal consumers are "locked in." As SC rise, fewer customers switch and demand for each product becomes inelastic. This sorting tends to encourage an increase in price as harvesting comes to dominate compensating: It becomes very hard to attract new customers with discounts, while loyal customers are less likely to leave due to prices increases. When SC are not very high, the dynamic effect encourages investing in shares through lower prices, partially offsetting the growing impact of the harvesting effect. As the market becomes more segmented, the incentive to "harvest" grows while the incentive to attract non-loyal customers (i.e., invest in share) diminishes. Eventually, firms begin to raise prices when facing high SC. As panel B shows, the monopolist's steady-state price increases with an increase in SC only if the SC effectively segmented the market based on customer loyalty. Steady-state prices and shares behave very similarly to the monopoly case, but competition shifts prices and profits down. This suggests that the effect of SC on market outcomes depends primarily on the existence of an alternative product with similar SC regardless of whether that alternative product adjusts prices strategically. Overall, the message of this figure is that the marginal effects of SC identified in the single-firm analysis carry to multi-firm analysis.

Lastly, figure 4 illustrates the effect of market power by comparing a 'moderate' monopoly (the full line, $\delta_1 = 1$) with a 'powerful' monopoly (the dotted line, $\delta_1 = 3$). As in the previous figures, the outside good's value is equal to the cost $(\delta_0 = c = 0)$.[32] The figure shows that effect of moderate SC on steady-state profits is

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[32]We have experimented with alternative specifications and found the results to be robust and indicative of the key insights. The upper bound of $\bar{\gamma} = 2$ reflects the upper bound for the steady state to exist. Analysis available from the authors shows that the true upper bound is slightly above $\bar{\gamma} = 2$ and decreases with δ. Reducing δ (i.e. making the market more competitive) allows increasing $\bar{\gamma}$

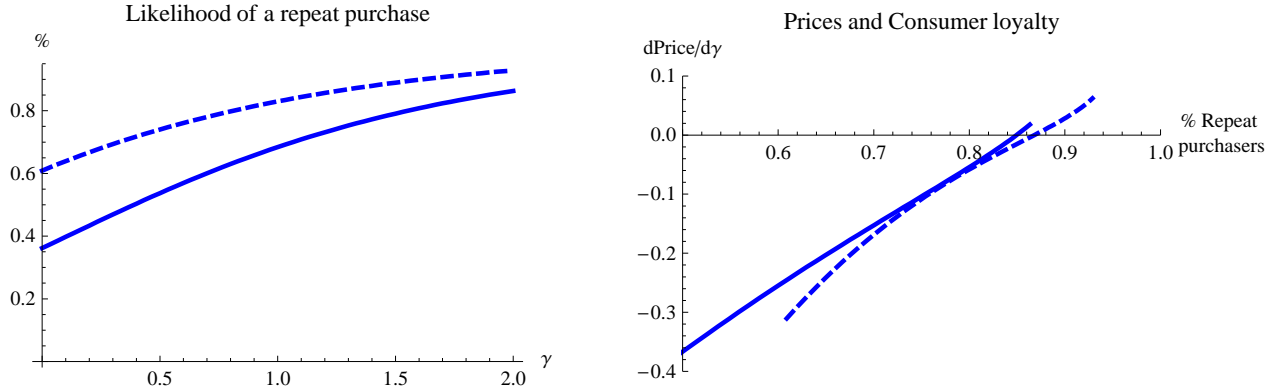"Consumer Loyalty and the Price Effect of Switching Costs"

Figure 3. Panel A shows the market segmentation as a function of switching costs. The proportion of consumers in each period who make repeat purchases is plotted for the different values of switching costs. As switching costs grow, consumers sort themselves by loyalty group and demand for the product becomes inelastic, with customers likely to buy the good they are loyal to. Panel B shows the relation between market segmentation and the effect of switching costs on prices. The difference in steady-state prices as switching costs increase is plotted as a function of the share of consumers who make repeat purchases in the steady state. Both plots compare the effect for the low-quality monopoly ($\delta_1 = 1$, solid line) and high-quality monopoly ($\delta_1 = 3$, dashed line). Note that the curves nearly merge when a repeat purchase is very likely. While in both cases the firm's steady state price increases with SC only when a repeat purchase is almost guaranteed, the strong monopolist requires a larger share of repeat purchases. Parametrizations for this plot are $\alpha = -1$, $\delta_0 = 0$, $\beta = 0.95$

consistent with the analytic results for low SC. The strong firm's profits increase while the weaker firm's profits decrease. As SC increase, the effect on profits seems to level off. As long as the dominating static effect is the compensation effect (i.e. the "normal" monopolist), the firm's profits are highest without SC. This example also illustrates that the strong firm gains from the introduction of SC even though it's optimal reaction is to charge lower prices.

4. Conclusion

SC are an important part of many markets. In this paper we have shown that if firms cannot price discriminate between loyal and non-loyal consumers, firm behavior is shaped by three forces: (1) harvesting surplus from loyal customers who must pay SC to purchase other goods, (2) compensating non-loyal consumers to convince them to switch, and (3) investing in share in order to increase the number of loyal consumers in future periods. The first two effects are due to present-day incentives while the final one is due to the incentive to generate future profits. The impact of SC has frequently been described as pitting a short term incentive to raise prices against a long term incentive to maintain a high share (i.e., harvesting versus investing). However, we have shown that

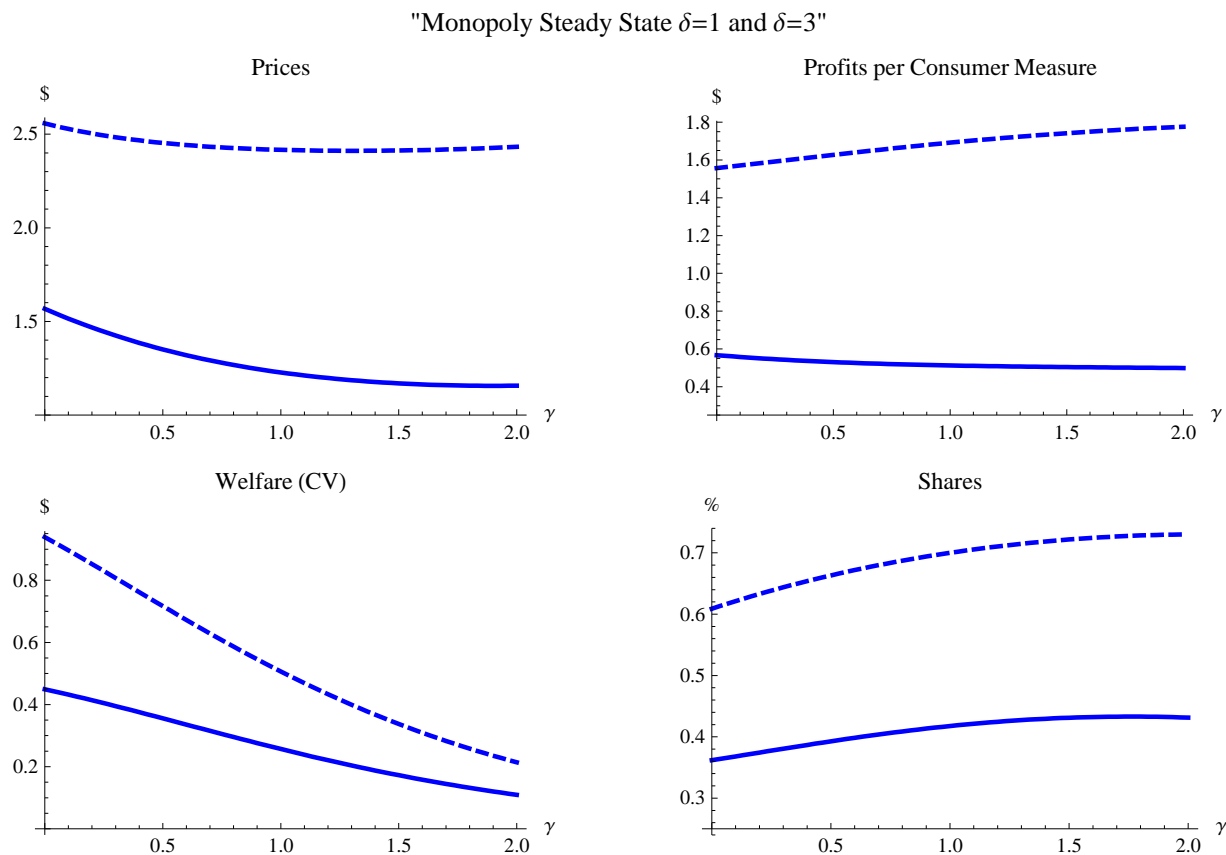## "Monopoly Steady State $\delta=1$ and $\delta=3$"

FIGURE 4. Steady-state outcomes for a monopolist with respect to switching costs in the logit model. The dashed line represents the high-quality monopoly ($\delta_1 = 3$). The full line represents the low-quality monopoly ($\delta_1 = 1$). The switching-costs value (γ) varies on the x-axis from zero to two. The other parameters are: $\delta_0 = 0, \alpha = -1, \beta = 0.95$.

in many cases the compensating effect is substantial. Indeed, it may overwhelm the harvesting effect such that the overall short term incentives are to decrease prices, without even considering the investment incentive.

Of the three forces related to SC, only one—harvesting—is anti-competitive. SC are anti-competitive if the market is segmented so that harvesting dominates firms' short term incentives. Segmentation can arise from very high SC or because of reasons that are independent of SC such as substantial quality differences between firms (considered explicitly in the current model), network externalities or persistent consumer preferences. In either case, such frictions increase the fraction of consumers that are, to a varying extent, "locked-in" to their previous purchases.

While analytic results can only be obtained for small SC, the economic intuitions persisted in numerical experiments for large SC that cause 80 percent of consumers to be repeat purchasers. The empirical evidence cited at the introduction suggests that this is the relevant range for SC in many markets. Hence the compensating

effect we highlight, which is masked by assumptions such as price-discrimination or infinite SC, is likely have a significant impact in a number of retail industries.

An important conclusion for policy and empirical work is that changes in prices or shares as a result of SC may not be a valid proxy for changes in welfare or profits. Even if consumers pay lower prices with SC, they suffer welfare loses directly due to paying the switching cost and indirectly because they may consume a lower-utility product to avoid paying the switching cost. The examples show that the introduction of SC may cause declines in prices, profits and consumer welfare. In other words, one should not assume that a program to reduce SC is a failure simply because it does not cause price declines.

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Appendix A. Proofs

A.1. **Proof of Lemma 2.1.** The derivative of firm j's steady-state value with respect to SC is given by

$$\frac{dV^j}{d\gamma} = \frac{p_j^*}{1-\beta} \cdot \left[D_\gamma^j + \sum_{k \neq j} \left(\frac{dp_k^*}{d\gamma} \cdot D_{p_k}^j \right) \right],$$

where p^* represents the vector of steady state prices for the equilibrium when $\gamma = 0$.

Proof. For $\frac{dV^j}{d\gamma}$: by construction, in any steady state equilibrium

$$V^j = \frac{p_j D^j}{1 - \beta}$$

Apply the envelope theorem ($\frac{dV^j}{dp_j} = 0$) to get

$$\frac{dV^j}{d\gamma} = \frac{p_j}{1 - \beta} \frac{dD^j}{d\gamma}$$

Using

$$D^j = \sum_i s_i D^{j,i} \, ,$$

$$\frac{dD^j}{d\gamma} = \sum_i s_i \frac{dD^{j,i}}{d\gamma} = s_j D_\gamma^{j,j} + \sum_{i \neq j} \left(s_i D_\gamma^{j,i} + \frac{dp_i}{d\gamma} D_{p_i}^j \right)$$

Which simplifies to the desired result. $\square$

A.2. **Proof of Lemma 2.2.** The derivative of firm j's steady state price with respect to SC at $\gamma = 0$ has the same sign as F_γ^j, given by equation 2.6 (repeated here):

$$F_{\gamma=0}^j = D_\gamma^j + p_j^* \cdot D_{p_j \gamma}^j + \beta p_j^* \cdot \sum_{k \in J} \left(D_\gamma^{j,k} \cdot D_{p_j}^k \right) + \sum_{k \neq j} \left(\frac{dp_k^*}{d\gamma} \chi_{jk} \right)$$

Where all $\chi_{jk} \geq 0$ and p^* represent the vector of steady state prices for the equilibrium when $\gamma = 0$. Moreover, long run incentives—i.e., $\sum_{k \in J} \left(D_\gamma^{j,k} \cdot D_{p_j}^k \right)$—are always negative.

Proof. For F_γ^j: first, derive $\frac{dV}{d\nabla_{p_j}}$ by applying the envelope theorem to $V(s) = p D^1 + \beta V(s)$:

$$\frac{dV^j}{d\nabla} = p_j \cdot \frac{dD^j}{d\nabla} + \beta \cdot \frac{dV^j}{dD} \cdot \frac{dD}{d\nabla}$$

Where dD is the gradient of the change in demand for all firms that resulted from the change in the started shares ∇. Next, take the total derivative with respect to λ,

$$\frac{d\frac{dV^j}{d\nabla}}{d\gamma} = \frac{d\left(p_j \cdot \frac{dD^j}{d\nabla} \right)}{d\gamma} + \beta \cdot \left(\frac{d\frac{dV^j}{dD}}{d\gamma} \cdot \frac{dD}{d\nabla} + \frac{dV^j}{dD} \cdot \frac{d\frac{dD}{d\nabla}}{d\gamma} \right) \cdot$$

At $\gamma = 0$, $\frac{dV^j}{dD} = \frac{dD}{d\nabla} = 0$ so the second dynamic term is zero. As we are holding price fixed, at $\gamma = 0$,

$$\frac{d\frac{dV^j}{d\nabla}}{d\gamma = 0} = \frac{d\left(p_j \cdot \frac{dD^j}{d\nabla} \right)}{d\gamma} = p_j \cdot \frac{d\frac{dD^j}{d\nabla}}{d\gamma}$$

Therefore

$$F_{\gamma=0}^j = D_\gamma^j + p_j D_{\gamma p_j}^j + \beta \cdot p_j \cdot \left(\frac{d \frac{dD^j}{d\nabla_{p_j}}}{d\gamma} \otimes \frac{d\nabla_{p_j}}{dp_j} \right) + \sum_{i \neq j} \left(\frac{dp_i^*}{d\gamma} \chi_{j,i} \right)$$

It remains to simplify the product multiplication. $\frac{dD^j}{d\nabla_{p_j}}$ is a vector with elements $D_{s_i}^j$. By construction

$$D^j = \sum_i s_i D^{j,i}$$

At $\gamma = 0$, $D_{s_i}^{j,i} = 0$ and so

$$D_{s_i}^j = D^{j,i} \quad .$$

This identifies the elements of $\frac{dD^j}{d\nabla_{p_j}}$. Taking the derivative w.r.t. γ on the vector $\frac{dD^j}{d\nabla_{p_j}}$ yields elements of the form

$$\frac{d \frac{dD^j}{d\nabla_{p_j}}}{d\gamma} = \left\{ D_\gamma^{j,i} \right\} \quad .$$

Now perform the product multiplication:

$$\left(\frac{d \frac{dD^j}{d\nabla_{p_j}}}{d\gamma} \otimes \frac{d\nabla_{p_j}}{dp_j} \right) = \sum_{i \in J} D_\gamma^{j,i} \frac{\partial D^i}{\partial p_j}$$

Placing in F_γ^j above obtains the desired result:

$$F_{\gamma=0}^j = D_\gamma^j + p_j D_{\gamma p_j}^j + \beta \cdot p_j \cdot \left(\sum_{i \in J} D_\gamma^{j,i} \frac{\partial D^i}{\partial p_j} \right) + \sum_{i \neq j} \left(\frac{dp_i^*}{d\gamma} \chi_{j,i} \right)$$

To see that long run incentives are always negative, observe that $D_\gamma^{j,k}$ is positive iff $k = j$, while $D_{p_j}^k$ is negative iff $k = j$.

$\square$

A.3. **Proof of Lemma 3.1.** With logit demand, at $\gamma = 0$:

$$D_\gamma^j = s_j \cdot (s_j - H)$$

$$F_\gamma^j = \frac{s_j^2}{1 - s_j} \cdot \left((s_j - H) - \beta \cdot (1 + H - 2s_j) \right) + \sum_{k \neq j} \left(\frac{dp_k^*}{d\gamma} \chi_{jk} \right)$$

Where all $\chi_{jk} \geq 0$.

Proof. From Lemma 2.2 we know that,

$$F_\gamma^j = D_\gamma^j + p_j D_{\gamma p_j}^j + \beta \cdot p_j \cdot \left(\sum_{i \in J} D_\gamma^{j,i} \frac{\partial D^i}{\partial p_j} \right) + \sum_{k \neq j} \left(\frac{dp_k^*}{d\gamma} \chi_{jk} \right) .$$

So we can simply uses the structure of logit demand to determine each element in this equation to arrive at the second expression in the lemma. Note that the first expression of the lemma is shown in step 5 below.

(1) Determine $\frac{dD^i}{dp_j}$:

- For $i = j$, the element is,

$$\frac{\partial D^j}{\partial p_j} = -\alpha D^j \left(1 - D^j\right) = -\alpha s_j \left(1 - s_j\right).$$

- For $i \neq j$, the element is,

$$\frac{\partial D^i}{\partial p_j} = \alpha D^j D^i = \alpha s_j s_i.$$

- Observe that the irrelevant alternatives (IIA) assumption for how shares compensate is maintained,

$$\frac{\partial D^i}{\partial p_j} = \frac{\partial D^j}{\partial p_j} \cdot \left(-\frac{s_i}{1 - s_j}\right).$$

(2) Determine $D_\gamma^{j,i}$:

- For $i = j$, $D_\gamma^{j,j} = s_j \left(1 - s_j\right)$;
- For $i \neq j$, $D_\gamma^{j,i} = -s_j s_i$.

(3) Determine the products,

$$D_\gamma^{j,j} \cdot \frac{\partial D^j}{\partial p_j} = -\alpha s_j^2 \left(1 - s_j\right)^2;$$

$$D_\gamma^{j,i} \cdot \frac{\partial D^i}{\partial p_j} = -\alpha s_j^2 s_i^2.$$

(4) The sum,

$$\beta \cdot p_j \cdot \left(\sum_{i \in J} D_\gamma^{j,i} \frac{\partial D^i}{\partial p_j}\right) = -\beta \alpha p_j s_j^2 \left((1 - s_j)^2 + \sum_{i \neq j} s_i^2\right)$$

$$= -\beta \alpha p_j s_j^2 \left(1 - 2s_j + H\right).$$

Now use the standard first order condition at $\gamma = 0$,

$$\alpha p_j s_j \left(1 - s_j\right) = s_j.$$

So $\alpha p_j = \frac{1}{1 - s_j}$ and the sum is,

(A.1) $$\beta \cdot p_j \cdot \left(\sum_{i \in J} D_\gamma^{j,i} \frac{\partial D^i}{\partial p_j}\right) = -\beta \frac{s_j^2}{1 - s_j} \left(1 - 2s_j + H\right).$$

(5) Determine D_γ^j,

$$D_\gamma^j = \sum_i s_i D_\gamma^{j,i}$$

$$= s_j s_j (1 - s_j) - \sum_{i \neq j} s_i s_i s_j$$

$$= s_j^2 - s_j s_j^2 - s_j \sum_{i \neq j} s_i^2$$

$$= s_j^2 - s_j \cdot \sum_{i \in J} s_i^2$$

$$= s_j (s_j - H).$$

This is the first statement in the lemma, but it still remains to derive the price effect.

(6) Determine $D_{p_j \gamma}^j$,

$$D_{\gamma p_j}^{j,j} = D_{p_j}^{j,j} (1 - D^{j,j}) - D^{j,j} D_{p_j}^{j,j}$$

$$= D_{p_j}^{j,j} (1 - 2D^{j,j})$$

$$= -\alpha s_j (1 - s_j) (1 - 2s_j)$$

$$D_{\gamma p_j}^{j,i \neq j} = -D_{p_j}^{j,i} D^{i,i} - D^{j,i} D_{p_j}^{i,i}$$

$$= \alpha D^{j,i} (1 - D^{j,i}) D^{i,i} - \alpha D^{j,i} D^{j,i} D^{i,i}$$

$$= \alpha [s_j (1 - s_j) s_i - s_j s_s s_i]$$

$$= \alpha s_j s_i (1 - 2s_j)$$

$$D_{\gamma p_j}^j = \sum_i s_i D_{\gamma p_j}^{j,i}$$

$$= -\alpha s_j^2 (1 - s_j) (1 - 2s_j) + \sum_{i \neq j} \alpha s_j s_i^2 (1 - 2s_j)$$

$$= \alpha s_j (1 - 2s_j) \cdot \left(\sum_{i \neq j} s_i^2 - s_j + s_j^2 \right)$$

$$= \alpha s_j (1 - 2s_j) (H - s_j).$$

Using $\alpha \cdot p_j = \frac{1}{1-s_j}$ (again, from the first order condition at $\gamma = 0$):

$$p_j D^j_{\gamma p_j} = \frac{s_j}{1 - s_j} \left(1 - 2s_j\right) \left(H - s_j\right).$$

(7) Combine these to determine the price effect:

$$F^j_\gamma = D^j_\gamma + p_j D^j_{\gamma p_j} + \beta \cdot p_j \cdot \left(\sum_{i \in J} D^{j,i}_\gamma \frac{\partial D^i}{\partial p_j}\right) + \sum_{k \neq j} \left(\frac{dp^*_k}{d\gamma} \chi_{jk}\right)$$

$$= s_j \left(s_j - H\right) + \frac{s_j}{1 - s_j} \left(1 - 2s_j\right) \left(H - s_j\right) - \beta \frac{s_j^2}{1 - s_j} \left(1 - 2s_j + H\right) + \sum_{k \neq j} \left(\frac{dp^*_k}{d\gamma} \chi_{jk}\right)$$

$$= \frac{s_j}{1 - s_j} \left[\left(s_j - H\right)\left(1 - s_j\right) + \left(H - s_j\right)\left(1 - 2s_j\right) - \beta s_j \left(1 - 2s_j + H\right)\right] + \sum_{k \neq j} \left(\frac{dp^*_k}{d\gamma} \chi_{jk}\right)$$

$$= \frac{s_j}{1 - s_j} \left[\left(H - s_j\right)\left(1 - 2s_j - 1 + s_j\right) - \beta \left(1 - 2s_j + H\right)\right] + \sum_{k \neq j} \left(\frac{dp^*_k}{d\gamma} \chi_{jk}\right)$$

$$= \frac{s_j^2}{1 - s_j} \left[-\left(H - s_j\right) - \beta \left(1 - 2s_j + H\right)\right] + \sum_{k \neq j} \left(\frac{dp^*_k}{d\gamma} \chi_{jk}\right).$$

Which is the second statement in the lemma.

$\square$

A.4. **Proof of Proposition 3.1.** In a random utility model with quasi-linear utility, if firms are symmetric $(\delta_j = \delta)$, then:

(1) If firms are myopic $(\beta = 0)$, small SC have no effect on prices and profits.
(2) If firms are forward looking $(\beta > 0)$, prices and profits decrease from a small SC for all firms.

Proof. With quasi-linear utility and J symmetric firms, consumers that are loyal to firm 1 purchase the firm's product if

$$-\alpha p_1 - \varepsilon_1 + \gamma > \max_{j \neq 1} \left\{-\alpha p_j - \varepsilon_j\right\}.$$

As all ε_j are iid, this share can be described by a CDF on the first order statistic of the ε_j distribution and the rival's prices:

$$D^{1,1} = \Phi \left(K - \alpha p_1 + \gamma\right),$$

Where K is a (linear) function of all the firms' rival prices. Letting ϕ denote the PDF for Φ:

$$D^{1,1}_\gamma = \phi\left(K - \alpha p_1 + \gamma\right) \quad \text{and at } \gamma = 0: \quad D^{1,1}_\gamma = \phi\left(K - \alpha p_1\right),$$

$$D^{1,1}_{p_1} = -\alpha \cdot \phi\left(K - \alpha p_1 + \gamma\right) \quad \text{and at } \gamma = 0: \quad D^{1,1}_{p_1} = -\alpha \cdot \phi\left(K - \alpha p_1\right),$$

$$D^{1,1}_{\gamma p_1} = -\alpha \cdot \phi'\left(K - \alpha p_1 + \gamma\right) \quad \text{and at } \gamma = 0: \quad D^{1,1}_{\gamma p_1} = -\alpha \cdot \phi'\left(K - \alpha p_1\right).$$

Note that as $D^{1,1}_{p_1} < 0$ and $\alpha > 0$ it must be that $\phi\left(K - \alpha p_1\right) > 0$.

Consumers that are loyal to firm $k \neq 1$, would purchase if

$$-\alpha p_1 - \varepsilon_1 > \max\left\{\max_{j \neq 1,k}\left\{-\alpha p_j - \varepsilon_j\right\}, -\alpha p_k + \gamma - \varepsilon_k\right\}$$

Because utility is quasi-linear and the shocks are independent, this share can be described by a CDF on the first order statistic of the ε distribution, the rivals' prices and a function $\tau\left(\gamma\right)$ that depends only on γ:

$$D^{1,k} = \Phi\left(K - \alpha p_1 - \tau\left(\gamma\right)\right).$$

Let $\tau_0 \equiv \tau'\left(0\right)$. The partials are:

$$D^{1,k}_\gamma = -\tau'\left(\gamma\right)\phi\left(K - \alpha p_1 - \tau\left(\gamma\right)\right) \quad \text{and at } \gamma = 0: \quad D^{1,k}_\gamma = -\tau_0 \cdot \phi\left(K - \alpha p_1\right),$$

$$D^{1,k}_{p_1} = -\alpha \cdot \phi\left(K - \alpha p_1 - \tau\left(\gamma\right)\right) \quad \text{and at } \gamma = 0: \quad D^{1,k}_{p_1} = -\alpha \cdot \phi\left(K - \alpha p_1\right),$$

$$D^{1,k}_{\gamma p_1} = \tau'\left(\gamma\right)\alpha \cdot \phi'\left(K - \alpha p_1 - \tau\left(\gamma\right)\right) \quad \text{and at } \gamma = 0: \quad D^{1,k}_{\gamma p_1} = \tau_0 \cdot \alpha \cdot \phi'\left(K - \alpha p_1\right).$$

We now prove that, at $\gamma = 0$, if demand is quasi-linear, and the market is symmetric and fully covered (no outside good), then $\tau_0 = \frac{1}{J-1}$ and $D^1_{\gamma p_1} = 0$.

By construction, at $\gamma = 0$,

$$D^1_\gamma = \frac{D^{1,1}_\gamma + \left(J - 1\right)D^{1,k}_\gamma}{J} = \frac{\phi\left(K - \alpha p_1\right) - \left(J - 1\right)\tau_0\phi\left(K - \alpha p_1\right)}{J}$$

$$= \frac{\phi\left(K - \alpha p_1\right)}{J}\left(1 - \tau_0\left(J - 1\right)\right),$$

$$D^1_{\gamma p_1} = \frac{D^{1,1}_{\gamma p_1} + \left(J - 1\right)D^{1,k}_{\gamma p_1}}{J} = \frac{-\alpha \cdot \phi'\left(K - \alpha p_1\right) + \left(J - 1\right)\tau_0\alpha \cdot \phi'\left(K - \alpha p_1\right)}{J}$$

$$= \frac{\alpha \cdot \phi'\left(K - \alpha p_1\right)}{J}\left(\tau_0\left(J - 1\right) - 1\right).$$

By symmetry, if there's no outside good, $D_\gamma^1 = 0$, since steady state shares of all firms must remain $\frac{1}{J}$ with and without switching costs. As $\phi(K - \alpha p_1) \neq 0$, this requires that,

$$1 = \tau_0 (J - 1)$$

which proves both results.

This proves that the short term effect is zero and the price reaction depends only on the long term investment effect.

By Lemma 2.2, the investment effect only decreases prices and by strategic complementarity and symmetry prices decrease for all firms. As quantity sold does not change and prices decrease, profits decrease as well.

$\square$

A.5. **Proof of Proposition 3.2.** In a symmetric logit demand model with SC, if firms are forward looking $(\beta > 0)$, the effect of SC on prices strictly decreases in absolute terms when a firm is added to the market. The result applies to all quasi-linear random utility models in which equilibrium price sensitivity $(D_{p_j}^j)$ weakly decreases in absolute terms when a firm is added to the market.

Proof. From Proposition 3.1, it is sufficient to show that the claim is true for the investment effect. The investment effect is $\beta \cdot p_1 \cdot \sum_i \left(D_\gamma^{1,i} D_{p_1}^i \right)$

The proof of Proposition 3.1, showed that at $\gamma = 0$, $D_{p_1}^{1,1} = D_{p_1}^{1,j} = D_{p_1}^1$. Symmetry implies that $1's$ gain (or loss) is spread evenly over it's rivals:

$$D_{p_1}^j = -\frac{D_{p_1}^1}{J - 1}.$$

In addition, applying the notation and results from the proof of Proposition 3.1, at $\gamma = 0$, we have that $\tau_0 = \frac{1}{J-1}$ and:

$$D_\gamma^{1,j} = -\frac{D_\gamma^{1,1}}{J - 1}.$$

Thus,

$$D_\gamma^{1,j} D_{p_1}^j = \left(-\frac{D_\gamma^{1,1}}{J - 1} \right) \cdot \left(-\frac{D_{p_1}^1}{J - 1} \right) = \frac{D_\gamma^{1,1} D_{p_1}^1}{(J - 1)^2}.$$

Therefore,

$$\sum_i \left(D_\gamma^{1,i} D_{p_1}^i \right) = (J - 1) \frac{D_\gamma^{1,1} D_{p_1}^1}{(J - 1)^2} + D_\gamma^{1,1} D_{p_1}^1$$

$$= D_\gamma^{1,1} D_{p_1}^1 \cdot \left(\frac{1}{J - 1} + 1 \right)$$

$$= D_\gamma^{1,1} D_{p_1}^1 \cdot \frac{J}{J - 1}.$$

So the investment effect is,

$$\beta \cdot p_1 D_\gamma^{1,1} D_{p_1}^1 \frac{J}{J-1}.$$

In any equilibrium at $\gamma = 0$, the first order condition on price implies $p_1 D_{p_1}^1 = -s_1$ and in the symmetric case this is $-\frac{1}{J}$ so we obtain

$$F_\gamma^1 = -\beta D_\gamma^{1,1} \frac{J}{J(J-1)} = -\frac{\beta D_\gamma^{1,1}}{J-1} = -\frac{\beta}{J-1}\phi(K-\alpha p_1).$$

An increase in J increases the denominator. Thus, it is sufficient to show that an increase in J weakly decreases the numerator. As $D_{p_1}^1 = -\alpha \cdot \phi(K-\alpha p_1)$, this is the condition stated in the proposition for the general quasi-linear random utility case. For the special case of the logit distribution, in any equilibrium:

$$\phi(K-\alpha p_1) = \frac{J-1}{J^2}.$$

And therefore,

$$F_\gamma^1 = -\beta \frac{J-1}{J^2} \frac{1}{(J-1)} = -\beta \frac{1}{J^2}.$$

The same investment effect can be obtained for the logit directly by plugging in $s_j = \frac{1}{J}$ and $H = \frac{1}{J}$ into the investment effect for the logit model (3.5).

$\square$

A.6. **Proof of Proposition 3.3.** Suppose in the logit demand model, $J \geq 2$, firm 1 has share $s_1 > \frac{1}{J}$ and all other firms have equal shares $s_{j>1} = \frac{1-s_1}{J-1}$. A sufficient condition for all prices to decrease with SC is,

$$(A.2) \qquad\qquad s_1 \leq \frac{1+J\beta}{J+J\beta}.$$

Proof. Start with deriving H:

$$H = s_1^2 + \left(\frac{1-s_1}{J-1}\right)^2 \cdot (J-1) = s_1^2 + \frac{(1-s_1)^2}{J-1}$$

By strategic complementarity, to prove that all prices decline, it is sufficient to show that for the strong firm $(j=1)$, $F_\gamma^1 < 0$. From (3.5) we see that this condition is,

$$-(H-s_1) - \beta(1-2s_1+H) < 0.$$

Placing H and simplifying, the condition is,

$$\frac{1 - s_1}{J - 1} \left(Js_1 - 1\right) - \beta \left(1 + s_1^2 + \frac{(1 - s_1)^2}{J - 1} - 2s_1\right) < 0.$$

Simplifying further, the condition is

$$Js_1 - 1 - J\beta \left(1 - s_1\right) < 0$$

Isolating s_1 obtains the statement in the proposition. $\qquad\square$

A.7. **Proof of Proposition 3.4.** In the logit model, if all firms are symmetric, $\delta_1 = \delta_2 = \cdots = \delta_{J-1}$, and an outside good has a share s_J, then:

(1) Prices and profits **increase** with a small SC if firms are myopic ($\beta = 0$) and $s_J < \frac{1}{J}$.

(2) Prices decrease from a small SC whenever $s_J > \frac{1}{J}$ or $\beta \geq 0.077$

Proof. In this setting, $s_j = \frac{1 - s_J}{J - 1}$ and

$$H = s_j \left(1 + \frac{s_J \left(J \cdot s_J - 1\right)}{1 - s_J}\right)$$

$$H - s_j = \frac{s_j s_J}{1 - s_J} \left(J \cdot s_J - 1\right) = \frac{s_J}{J - 1} \left(Js_J - 1\right)$$

Thus, a sufficient condition for profits to increase is that $s_J < \frac{1}{J}$ (as this implies $s_j - H > 0$) and prices do not decrease. Recalling the price effect formula (3.5) at $\gamma = 0$,[33]

$$F_\gamma^j = \frac{s_j^2}{1 - s_j} \left[-\left(H - s_j\right) - \beta \left(1 - 2s_j + H\right)\right]$$

A sufficient condition for the price effect to be positive is that $\beta = 0$ and $s_j - H > 0$. Thus, profits increase with a small SC if $s_J < \frac{1}{J}$.

The sign of the price effect is determined by

$$F_\gamma^j \gtrless 0 \iff -\left(H - s_j\right) - \beta \left(1 - 2s_j + H\right) \gtrless 0$$

The second term, which is subtracted, is always positive:

$$1 - 2s_j + H = 1 - s_j + H - s_j = 1 - s_j + s_j \left(s_j - 1\right) + \sum_{i \neq j} s_i^2$$

$$= \left(1 - s_j\right)^2 + \sum_{i \neq j} s_i^2 \geq 0$$

---

[33]Here, because all price-setting firms are symmetric and the game is in strategic complements, can drop the strategic interaction term because we know it will only re-enforce the incentives of the leading terms.

Therefore, a sufficient condition for price to decrease is that $H - s_j \geq 0$, which, from above, is equivalent to $s_J \geq \frac{1}{J}$.

If $s_J \leq \frac{1}{J}$ then $s_j - H \geq 0$ and,

$$F_\gamma^j \gtreqless 0 \iff s_j - H \gtreqless \beta \cdot (1 - 2s_j + H).$$

Isolating β, price decreases if,

$$\beta \geq \frac{s_j - H}{1 - s_j + H - s_j} = \frac{\frac{s_J}{J-1}(1 - Js_J)}{1 - \frac{1-s_J}{J-1} + \frac{s_J}{J-1}(Js_J - 1)}.$$

Simplifying:

$$\beta \geq \frac{s_J(1 - Js_J)}{J - 1 - 1 + s_J + s_J(Js_J - 1)}$$
$$\beta \geq \frac{s_J(1 - Js_J)}{J - 2 + Js_J^2}$$

Note that as $J \geq 3$, (the case of one firm and an outside good was handled in the previous section), the denominator is always positive. The derivative of the RHS for any s_j is,

$$\frac{-s_J^2(J - 2 + Js_j^2) - (1 + s_j^2)s_J(1 - Js_J)}{(J - 2 + Js_J^2)^2} \leq 0.$$

The sign follows from $s_J \leq \frac{1}{J}$ and $J \geq 3$. Thus, the strictest bound on β is for $J = 3$ is,

$$\beta \geq \frac{s_J - 3s_J^2}{1 + 3s_J^2}.$$

The worse case is when $s_J = \frac{2}{\sqrt{3}} - 1$ (≈ 0.155).[34] At this point, the lower bound on β is

$$\beta \geq \frac{1}{\sqrt{3}} - \frac{1}{2} \approx .07735.$$

Thus, for any $\beta \geq .0774$, prices are lower with a small SC than without. $\qquad\square$

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[34]The numerator for the derivative of the RHS wrt s_J is
$$(1 - 6s_J)(1 + 3s_J^2) - 6s_J(s_J - 3s_J^2) = 1 - 6s_J + 3s_J^2 - 18s_J^3 - 6s_J^2 + 18s_J^3$$
$$= 1 - 6s_J - 3s_J^2.$$
This quadratic expression is positive at $s_J = 0$ and negative at $s_J = \frac{1}{3}$ (the upper bound). So need to take the maximizing point,
$$s_J = \frac{-6 \pm \sqrt{36 + 12}}{6} = -1 + \frac{4\sqrt{3}}{6} = -1 + \frac{2}{\sqrt{3}}.$$

A.8. **Proof of Proposition 3.5.** For a monopolist:

$$\frac{dV^1}{d\gamma = 0} = \frac{p_1\,(2s_1 - 1)}{1 - \beta} D^{1,1}_\gamma$$

$$F^1_{\gamma=0} = (2s_1 - 1)\left(D^{1,1}_\gamma + p_1 D^{1,1}_{p_1\gamma}\right) - 2\beta s_1 \cdot D^{1,1}_\gamma$$

Thus:

(1) A small SC increases profits for a monopolist iff the monopolist share of the market is at least .5.

(2) For any discount factor a small SC causes a monopolist to decrease prices if its share is lower than .5.

(3) For any discount factor a small SC causes a monopolist to increase prices if in the equilibrium without SC: $2s_1 \cdot p_1 D^{1,1}_{p_1\gamma} > D^{1,1}_\gamma + p_1 D^{1,1}_{p_1\gamma}$.

Proof. We let the monopolist be firm 1 and the outside good firm 0. The following will be used throughout the proof:

Lemma A.1. *At* $\gamma = 0$:

(1) $D^{1,1} = D^{1,0}$

(2) $D^{1,1}_\gamma = -D^{1,0}_\gamma$

(3) $D^{1,1}_{p_1\gamma} = -D^{1,0}_{p_1\gamma}$

Proof. Recall that $D^{1,1} = \Phi\left(\delta - \alpha p_1 + \gamma\right)$ and $D^{1,0} = \Phi\left(\delta - \alpha p_1 - \gamma\right)$ The first claim follows from replacing $\gamma = 0$ in $D^{1,k}$. The second and third follow from taking the required derivatives and then placing $\gamma = 0$.   □

Applying Lemma 2.1:

$$\frac{dV^1}{d\gamma} = \frac{p^*}{1 - \beta} \cdot D^1_\gamma$$

By construction,

$$D^1_\gamma = s_1 \cdot D^{1,1}_\gamma + (1 - s_1) \cdot D^{1,0}_\gamma$$

Applying lemma (A.1):

$$D^1_\gamma = D^{1,1}_\gamma \cdot (2s_1 - 1)$$

To derive the price effect, first write F^1_γ for a monopolist:

$$F^1_\gamma = D^1_\gamma + p_1 D^1_{p_1\gamma} + \beta p_1 \cdot \left(D^{1,0}_\gamma D^0_{p_1} + D^{1,1}_\gamma D^1_{p_1}\right)$$

Applying lemma 2.1 for $D^{1,k}_\gamma$, D^1_γ and $D^1_{p_1\gamma}$ obtains

$$F^1_\gamma = D^{1,1}_\gamma \cdot (2s_1 - 1) + p_1 D^{1,1}_{p_1\gamma} \cdot (2s_1 - 1) + \beta p_1 \cdot 2 \cdot D^{1,1}_\gamma D^1_{p_1}$$

Collecting terms and recalling that at the steady state w/o SC $s_1 = -p_1 \cdot D^1_{p_1}$ obtains the desired F^1_γ.

The qualitative statements follow directly from these two equations:

(1) As $D^{1,1}_\gamma \geq 0$, the sign of the profit effect depends only on $2s_1 - 1$

(2) As $D^{1,1}_\gamma > 0$ and $(D^{1,1}_\gamma + p_1 \cdot D^{1,1}_{p_1\gamma} \geq 0$, if $2s_1 - 1 \leq 0$ it must be that $F^1_\gamma < 0$.

(3) As $D^{1,1}_\gamma \geq 0$, F^1_γ is lowest for $\beta = 1$. Placing $\beta = 1$ in F^1_γ yields,

$$F^1_\gamma(\beta = 1) = (2s_1 - 1)\left(D^{1,1}_\gamma + p_1 D^{1,1}_{p_1\gamma}\right) - 2s_1 \cdot D^{1,1}_\gamma.$$

Collecting terms obtains the stated result.

$\square$

A.9. **Proof of Proposition 3.6.** For a monopolist facing logit demand, the introduction of a small SC changes profits by

(A.3)
$$\frac{dV^1}{d\gamma = 0} = \frac{p_1 \cdot s_1 \cdot (1 - s_1) \cdot (2s_1 - 1)}{1 - \beta}.$$

The monopolist price increases iff $s_1 \geq \frac{1+2\beta}{2+2\beta}$.

Proof. The derivative of the value function with respect to switching costs can be found by substituting the logit expression for $D^{j,j}_\gamma = s_j(1 - s_j)$ into the general monopoly expression for $\frac{dV^1}{d\gamma=0}$ found in Proposition 3.5. To show that the monopolist price increases if and only if $s_1 \geq \frac{1+2\beta}{2+2\beta}$, first note that under monopoly,

$$H = s_1^2 + (1 - s_1)^2 = 2s_1^2 + 1 - 2s_1 = 1 - 2s_1(1 - s_1)$$

So

$$H - s_1 = (1 - s_1)(1 - 2s_1)$$

$$1 - 2s_1 + H = 2s_1^2 + 2 - 4s_1 = 2(1 - s_1)^2$$

Substituting this into the first order condition for prices,

$$F^1_\gamma = \frac{s_1^2}{1 - s_1} \cdot \left[(1 - s_1)(2s_1 - 1) - 2\beta(1 - s_1)^2\right]$$

$$= s_1^2 \cdot [2s_1 - 1 - 2\beta(1 - s_1)]$$

As $s_1 > 0$, the condition for prices to increase is

$$2s_1 - 1 - 2\beta(1 - s_1) \geq 0$$

Isolate β:

$$F_\gamma \geq 0 \iff \beta \leq \frac{1 - 2s_1}{2 - 2s_1}$$

Isolate s_1 to arrive at the expression in the Proposition:

$$s_1 \geq \frac{1 + 2\beta}{2 + 2\beta}$$

$\square$

Appendix B. Extensions

In this appendix, we consider two extensions to the basic model and illustrate how these affect the overall impact of switching costs. Formally, the effect of each extension can be determined by evaluating the change implied on the two basic comparative static equations (2.4) and (2.6). In particular, applying the framework used here, it is sufficient to evaluate the first order change in demand D_γ^j, in price sensitivity $D_{\gamma p_j}^j$ and in the dynamic value of investing in consumers $\beta \cdot D_\gamma^{j,k} \cdot D_{p_j}^k$.

B.1. **Time-Persistent Consumer Heterogeneity.** A natural variation of the model is to assume that the same consumers that preferred good j at period t would likely prefer good j at the following period. Intuitively, as the degree of horizontal differentiation between the goods increases, the marginal consumer for each firm is more likely a loyal repeat purchaser. As a result, the firms' short term incentive to compensate will be weaker and its incentive to harvest will be stronger, resulting in higher prices. Horizontal differentiation will also weaken the investing effect of SC – the market has less switchers to sway as more consumers choose based on their horizontal preference.

Thus, time-persistent horizontal differentiation should decrease the effects of SC that decrease profits (compensating) and prices (compensating and investing), while increasing the effect of SC that increase prices and profits (harvesting).

Formally, our derivation can be extended relatively easily to accommodate some types of time-persistent preferences. In particular, suppose that consumers are split at the start of the game into types, with ϕ_n the measure of type n in the general population. Letting $F_\gamma^{j,n}$ denote the first order effect of a low SC (i.e. equation 2.6) for firm j considering only consumers of type n, then the overall effect is simply,

(B.1) $$F_\gamma^j = \sum_n \phi_n \cdot F_\gamma^{j,n}.$$

To illustrate, consider the logit model with J symmetric firms and an outside good that has a lower than average market share (i.e., the outside share is less than $1/(J+1)$. Extend the model by allowing for $J + 1$ consumers

types. In particular, suppose that the measure of regular consumers is ϕ_0 while a consumer of type j assigns a very negative utility to purchasing any good other than good j or the outside good. That is, for ϕ_0 customers demand is as described in (3.3), while for ϕ_j of the consumers, firm j is a monopoly and demand is,

$$D^{j,k} = \frac{e^{\bar{u}_{j,k}}}{e^{\bar{u}_{0,k}} + e^{\bar{u}_{j,k}}}.$$

As ϕ_j increases, firm j's pricing is affected more by the monopolized consumers, whose decisions are completely independent of opponents' prices. In particular F_γ^j at $\gamma = 0$ increases with ϕ_j. As horizontal differentiation weakens competition even without considering SC. This analysis reinforces the intuition that SC should be more of a concern in industries in which competition is (or would be) weak even without SC.

B.2. **Heterogeneous SC.** The assumption that all customers have the same SC may not fit well in some industries. For example, Biglaiser, Cremer and Dobos (2013) suggest that SC between online music outlets differ across customers depending on the intensity of their use of the outlet.

For small SC, heterogeneous SC across consumers are similar to an even smaller SC *as long as the SC apply to all goods.* If only a fraction λ of consumers are affected by the switching cost, but this fraction is the same for all consumers regardless of their previous purchase, then D_γ^j, $D_{\gamma p_j}^j$ and $D_\gamma^{j,k} \cdot D_{p_j}^k$ are simply a fraction λ of the original homogeneous SC model. Thus, the qualitative results are unchanged, but the degree of the SC effect is smaller.

The uniformly heterogeneous SC setting also captures the effect of SC in "overlapping generations" models. In these models, a fraction λ of the consumers each period is replaced with new consumers that are not loyal to any of the goods. Economically and mathematically, the model is identical. A closely related variation is when consumers switch loyalty group only with some probability λ, and with the complementing probability retain their original loyalty, despite buying a good they are not loyal to. In this case the short run effect of SC is the same as in the basic model, while the long run investing effect is dampened by λ.

Biglaiser, Cremer and Dobos (2013) consider heterogeneous SC that affect only consumers of a subset of the goods in the market or only consumers that have a higher consumption value. First, suppose that SC only affect consumers switching out of good 1. In this case SC clearly increase 1's demand ($D_\gamma^1 > 0$) and decrease demand for all other goods ($D_\gamma^{j\neq 1} < 0$). Thus, the first order effect of SC on profits is, as expected, positive for firm 1 and negative for all others. However, even firm 1 may decrease prices, due to the investing effect. Indeed, with logit demand, it is easy to verify that firm 1 (and thus all firms) will decrease prices whenever $\frac{s_1}{1-s_1} \leq \beta$. In all

other cases, the first order effect of SC on firm 1 will be to increase prices. For all other firms, the first order effect of SC is to decrease prices.[35]

Appendix C. Salop Model

C.1. **Salop Demand.** This appendix considers the implications of a small SC on the Salop (1979) circular city model.[36] The results echo the qualitative results obtained using logit demand.

Formally, the J firms are evenly spaced on a circumference, with the firm location denoted x_j. Each consumer draws each period a position x_i on the circumference and pays a 'travel cost' of $c \cdot |x_j - x_i|$ if she chooses good j. As in the previous section, firm quality is denoted δ_j. Note that we do not require that the firms are symmetric. Each consumer i, loyal to good k chooses in each period t the good j that maximizes her period utility, which is given by

$$(\text{C.1}) \qquad u_{ijkt} = \delta_j + \alpha \cdot p_{jt} - \gamma \cdot \mathbf{1}\left[j \neq k\right] - c \cdot |x_j - x_i|$$

We assume that in the equilibrium without SC, all firms have a strictly positive market share. A direct implication of the linear structure is that there are no cross effects: $D^j_{\gamma p_j} = 0$. To illustrate the implication of SC in these settings, suppose that absent SC, a consumer is indifferent between goods 2 and 3. Add a small SC to the model, the consumer's preference just increases for the specific good j she bought in the previous period. If that good is 2, the consumer now strictly prefers good 2, increasing 2's equilibrium demand. The same applies for good 3. However, if the consumer is loyal to any other good, a sufficiently small SC does not affect it's preference. The share of consumers therefore whose SC are relevant for firm 2 are only those loyal to either firm 1, 2, or 3 at the previous period. In turn, these are given respectively by s_1, s_2 and s_3. The next result summarizes the implications for this setting.

Proposition C.1. *In the circular city model in which all firms have a strictly positive market share:*

- *A small SC has no effect if firms are symmetric $(s_j = s_i)$ and myopic $(\beta = 0)$*
- *If firms are symmetric and forward looking $(\beta > 0)$, the effect of SC strictly decreases in magnitude as the number of firms in the market increases.*
- *A small SC decreases prices if firms are sufficiently forward looking $\left(\beta \geq \frac{2}{3}\right)$, regardless of firms' relative shares.*

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[35]By first-order effect we mean that the analysis does not consider the firm's reaction to it's rival price change.

[36]A similar analysis can be applied to other models of linear demand, but the qualitative results are the same. In particular, both Rhodes (2014) and Shin et al. (2009) consider a Hotelling model. The Salop model was chosen to highlight the additional endogenous asymmetry caused by SC when there are more than two firms in the market.

- *A sufficient condition for a small SC to decrease prices is that firms are sufficiently symmetric and are somewhat forward looking. Formally:*

(C.2)
$$\forall j : s_j \leq \frac{s_{j-1} + s_{j+1}}{2 - 3 \cdot \beta}$$

Proposition C.1 provides the analogous results to Proposition 3.1 and describes the implications of asymmetry in the linear setting. If firms are symmetric, demand does not change with the SC and so the short term effect is always zero. Note that this applies to any model of linear demand with symmetric firms and a fully covered market. For the specific circular city model with symmetric firms, the relation in (C.2) is an equality when firms are myopic and a strict inequality otherwise.

The economic forces are the same as those identified in the previous sections. SC alter the incentives of the marginal consumers against switching. If a firm's share is larger than it's relevant rivals, the effect shifts the firm's demand up.

The return on investing in share is only relevant to those customers that in the next period would be marginal for the firm. As in the logit case, increasing the number of firms decreases the likelihood that a firm's marginal consumer in this period would also be marginal in the next period. As a result, the return on investment is lower and the effect of SC on prices is weaker.

If firms are sufficiently patient ($\beta \geq \frac{2}{3}$) prices always decrease with a small SC, regardless of shares. This is because the investment effect overpowers any possible short term gains from harvesting. Finally, to illustrate the relation in (C.2), observe that if in the equilibrium w/o SC no firm has a larger share than its two neighbors' combined share, a sufficient condition for prices to decrease is that $\beta \geq \frac{1}{3}$.

C.2. **Proof of Proposition C.1.** In the circular city model in which all firms have a strictly positive market share:

- A small SC has no effect if firms are symmetric ($s_j = s_i$) *and* myopic ($\beta = 0$)
- If firms are symmetric and forward looking ($\beta > 0$), the effect of SC strictly decreases in magnitude as the number of firms in the market increases.
- A small SC decreases prices if firms are sufficiently forward looking $\left(\beta \geq \frac{2}{3}\right)$, regardless of firms' relative shares.
- A sufficient condition for a small SC to decrease prices is that firms are sufficiently symmetric and are somewhat forward looking. Formally:

(C.3)
$$\forall j : s_j \leq \frac{s_{j-1} + s_{j+1}}{2 - 3 \cdot \beta}$$

Proof. For notational simplicity, define $D^j_{\delta_j} = m$. Then $D^j_{\delta_{j-1}} = D^j_{\delta_{j+1}} = -\frac{m}{2}$ and for all remaining firms $D^j_{\delta_i \neq j-1, j, j+1} = 0$. Therefore,

$$D^{j,j}_\gamma = m \ , \quad D^{j,j-1}_\gamma = D^{j,j+1}_\gamma = -\frac{m}{2}$$

and

$$D^j_\gamma = m \left(s_j - \frac{1}{2} s_{j-1} - \frac{1}{2} s_{j+1} \right).$$

For the investing effect we have,

$$D^{j,j}_\gamma \frac{\partial D^j}{\partial p_j} = -\alpha m$$

And for $i = j+1, j-1$

$$D^{j,i}_\gamma \frac{\partial D^i}{\partial p_j} = -\frac{m}{2} \cdot \frac{\alpha}{2} = -\frac{\alpha m}{4}$$

Using the first order condition at $\gamma = 0$, $p_j = \frac{s_j}{\alpha}$,

$$F^j_\gamma = m \cdot \left(s_j - \frac{1}{2} s_{j-1} - \frac{1}{2} s_{j+1} - \beta s_j \cdot \frac{3}{2} \right)$$

Thus:

(1) If firms are symmetric: $F^j_\gamma = -m \cdot \beta \cdot \frac{3}{2J}$. This proves the first two statements.

(2) If $\beta \geq \frac{2}{3}$ prices always decrease with SC.

(3) If $\beta < \frac{2}{3}$, prices always decrease if for all j

$$s_j \leq \frac{s_{j-1} + s_{j+1}}{2 - 3\beta}$$

$\square$